To write the equation of a line, you must be given one of the following sets of information:

1) the slope and the y-intercept

2) the slope and a point on the line

3) two points on the line

GIVEN SLOPE AND Y-INTERCEPT

Plug m and b in to slope-intercept form

Slope   y-int. $y = mx + b$

Write an equation for each of the following lines:

slope of $\dfrac{3}{2}$, y-intercept of -7

$$y = \frac{3}{2}x - 7$$

y-intercept of 4, slope of $-\dfrac{11}{18}$

$$y = \frac{-11}{18}x + 4$$

GIVEN SLOPE AND ANY POINT ON THE LINE

Point-Slope Form:
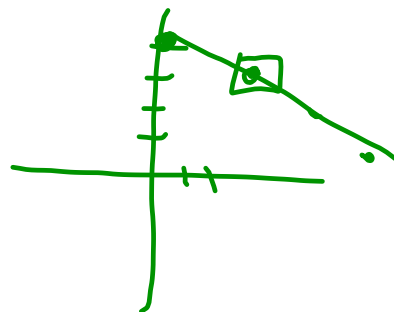
$$y - y_1 = m(x - x_1)$$

Steps:

1) Plug in slope for m

2) Plug in point for (x_1, y_1)

3) Solve for y

passes thru (2, 3) with a slope of $-\dfrac{1}{2}$

$$y - 3 = -\tfrac{1}{2}(x - 2)$$

$$y = -\tfrac{1}{2}x + 1 + 3$$

$$y = -\tfrac{1}{2}x + 4$$

passes thru (-3, 4) with a slope of $-\dfrac{2}{3}$

$$y - 4 = -\frac{2}{3}(x + 3)$$

$$y = -\frac{2}{3}x - 2 + 4$$

$$y = -\frac{2}{3}x + 2$$

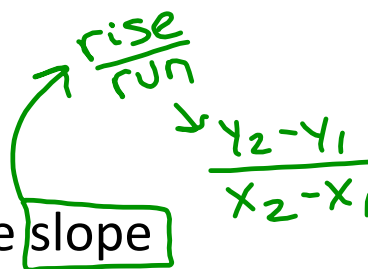passes thru (5, 6) with a slope of $\dfrac{2}{3}$

$$y - 6 = \frac{2}{3}(x - 5)$$

$$y = \frac{2}{3}x - \frac{10}{3} + 6$$

$$y = \frac{2}{3}x + \frac{8}{3}$$

GIVEN TWO POINTS ON THE LINE

Steps:

$\dfrac{\text{rise}}{\text{run}} \searrow \dfrac{y_2 - y_1}{x_2 - x_1}$

1) Use the given points to find the slope

2) Plug in either point, along with the slope from step 1, into the point-slope form

3) Solve for *y*

passing thru (-2, -1) and (3, 4)

$$m = \frac{-1-4}{-2-3} = \frac{-5}{-5} = 1$$

$$m = \frac{4-(-1)}{3-(-2)} = \frac{5}{5} = 1$$

$$y + 1 = 1(x + 2)$$

$$y = x + 2 - 1 \Rightarrow \boxed{y = x + 1}$$

$$y - 4 = 1(x - 3)$$

$$y = x - 3 + 4 \Rightarrow \boxed{y = x + 1}$$

passing thru (1, 5) and (4, 2)

$$m = \frac{5-2}{1-4} = \frac{3}{-3} = -1$$

$$y - 5 = -1(x-1)$$
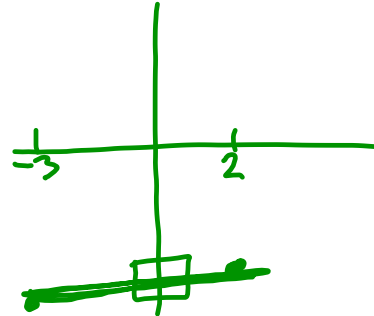
$$y = -x + 1 + 5 \implies \boxed{y = -x + 6}$$

passing thru (3, 0) and (-3, 1)

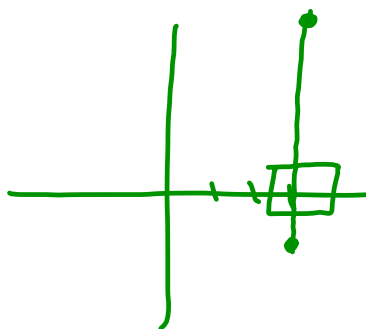$$y = -\frac{1}{6}x + \frac{1}{2}$$

passing thru (2, -5) and (-3, -5)
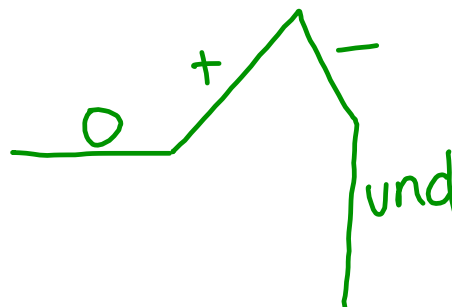
$$\frac{-5+5}{2+3} = \frac{0}{5} = 0$$

$$y = -5$$

passing thru (3, 7) and (3, -1)

$$\frac{7+1}{3-3} = \frac{8}{0} = und$$

$$x = 3$$

0   +   −

und

Two lines are PARALLEL if they lie in the same plane and never intersect

In order for two lines to never intersect, then they must be "rising" and "running" at the exact same rate.
Therefore, if two lines have equal slopes, then they will be parallel.

Two lines are PERPENDICULAR if they intersect to form a right angle

In order for two lines to intersect at a 90° angle, as one is "rising", then the other is "running" at the same rate, and vice versa. However, the directions in which they are moving will be different.
Therefore, if two lines have slopes that are negative reciprocals of each other, then they will be perpendicular.
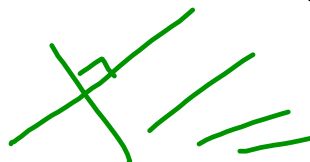
Determine which of the lines, if any, are parallel, and which are perpendicular

line a: through (-4, 1) and (-6, 7)

$$m = \frac{1-7}{-4+6} = \frac{-6}{2} = -3$$

line b: through (-7, -5) and (1, 11)

$$m = \frac{-5-11}{-7-1} = \frac{-16}{-8} = 2$$

line c: through (2, 5) and (4, 9)

$$m = \frac{5-9}{2-4} = \frac{-4}{-2} = 2$$

line d: through (4, 3) and (10, 5)

$$m = \frac{3-5}{4-10} = \frac{-2}{-6} = \frac{1}{3}$$

$$\therefore \; a \perp d$$
$$b \parallel c$$

passes thru (2, -3) and is <u>perpendicular</u> to the line $y = 2x - 3$

$$\perp m = -\frac{1}{2}$$

$$y + 3 = -\frac{1}{2}(x - 2)$$

$$y = -\frac{1}{2}x + 1 - 3$$

$$y = -\frac{1}{2}x - 2$$

passes through (-3, 4) and is parallel to $y = -\dfrac{2}{3}x + 7$

$// \quad m = -\dfrac{2}{3}$

$y - 4 = -\dfrac{2}{3}(x + 3)$

$y = -\dfrac{2}{3}x - 2 + 4$

$y = -\dfrac{2}{3}x + 2$

passes through (5, 6) and is parallel to $-2x + 3y = 11$

$y = \dfrac{2}{3}x + \dfrac{4}{3}$

$m = \dfrac{2}{3}$

$y - 6 = \dfrac{2}{3}(x - 5)$

$y = \dfrac{2}{3}x - \dfrac{10}{3} + 6$

$y = \dfrac{2}{3}x + \dfrac{8}{3}$

passes through (3, -5) and is <u>perpendicular</u> to the line through (1, 4) and (3, -2)

$$m = \frac{4+2}{1-3} = \frac{6}{-2} = -3$$

$$\perp m = \frac{1}{3}$$

$$y + 5 = \frac{1}{3}(x-3)$$

$$y = \frac{1}{3}x - 1 - 5$$

$$\boxed{y = \frac{1}{3}x - 6}$$

In $\triangle ABC$, $A(-3, 2)$, $B(9, 4)$, and $C(5, 12)$.
Write the equation of the median from C.

$$\text{midpt of } \overline{AB}: \left(\frac{-3+9}{2}, \frac{2+4}{2} \right)$$

$$= (3, 3)$$

$$m = \frac{12-3}{5-3} = \frac{9}{2}$$

$$y - 3 = \frac{9}{2}(x-3)$$

$$y = \frac{9}{2}x - \frac{27}{2} + 3$$

$$y = \frac{9}{2}x - \frac{21}{2}$$

In $\triangle ABC$, $A(-6, 2)$, $B(8, 4)$, and $C(18, 12)$.
Write the equation of the midsegment that is
parallel to $\overline{BC}$.

midpt $\overline{AB} = \left(\dfrac{-6+8}{2}, \dfrac{2+4}{2} \right)$

$= (1, 3)$

midpt $\overline{AC} : \left(\dfrac{-6+18}{2}, \dfrac{2+12}{2} \right)$

$= (6, 7)$

$m = \dfrac{3-7}{1-6} = \dfrac{-4}{-5} = \dfrac{4}{5}$

$y - 7 = \dfrac{4}{5}(x - 6)$

$y = \dfrac{4}{5}x - \dfrac{24}{5} + 7$

$y = \dfrac{4}{5}x + \dfrac{11}{5}$

In $\triangle ABC$, $A(2, 3)$, $B(12, 5)$, and $C(9, 8)$.
Write the equation of the perpendicular
bisector of $\overline{AB}$.

midpt $\overline{AB} : \left(\dfrac{2+12}{2}, \dfrac{3+5}{2} \right) = (7, 4)$

$m\ \overline{AB} = \dfrac{3-5}{2-12} = \dfrac{-2}{-10} = \dfrac{1}{5}$

$\perp m = -5$
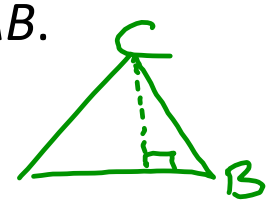
$y - 4 = -5(x - 7)$

$y = -5x + 35 + 4$

$y = -5x + 39$

In $\triangle ABC$, $A(-3, 4)$, $B(5, -2)$, and $C(-4, -5)$.
Write the equation of the altitude to $\overline{AB}$.

$$m\,\overline{AB} = \frac{4+2}{-3-5} = \frac{6}{-8} = \frac{-3}{4}$$

$$\perp m = \frac{4}{3}$$

$$y + 5 = \frac{4}{3}(x+4)$$

$$y = \frac{4}{3}x + \frac{16}{3} - 5$$

$$y = \frac{4}{3}x + \frac{1}{3}$$