

Combinations and Permutations

1. ${}_{12}P_4 = \frac{12!}{8!} = 11,880$

2. ${}_8P_5 = \frac{8!}{3!} = 6720$

3. ${}_{14}C_{11} = \frac{14!}{3!11!} = 364$

4. ${}_{(n+2)}^{(n+4)} = \frac{(n+4)!}{((n+4)-(n+2))!(n+2)!} = \frac{(n+4)(n+3)(n+2)!}{2!(n+2)!} = \frac{n^2 + 7n + 12}{2}$

5. You are ordering a pizza. The three-topping pizza costs \$8. If there are 18 toppings to choose from, 4 different crusts, and 2 different sauces, how many different 3-topping pizzas can you order?

$\frac{4C_1}{4} \cdot \frac{2C_1}{2} \cdot \frac{18C_3}{816} = 6528$

6. A radio station is playing 10 different songs. If 2 of the songs are going to be played twice, and one of the songs is going to be played 4 times, how many ways can the station fill 15 song slots?

repetition $\rightarrow \frac{15P_{15}}{2!2!4!} = 136,216,080,000$

*If we also need to decide 2 songs to repeat twice and 1 song to repeat 4 times: $10C_2 \cdot 8C_1$ or $10C_1 \cdot 9C_2$

7. Triangles are often labeled by placing a different letter at each vertex. In how many different ways could a given scalene triangle be labeled using any of the 26 letters of the alphabet?

${}_{26}P_3 = 15,600$

= 360
The massive answer would be multiplied by 360. Crazy, no?!

8. You are picking a hand of 5 cards from a standard deck of playing cards. What is the probability that your hand has 2 10s and 3 Jacks?

$\frac{4C_2}{6} \cdot \frac{4C_3}{4} = \frac{24}{2598960}$

2598960-5 card hands

9. At South High School, 55 students entered an essay contest. From these students, 10 are selected as finalists. How many different ten-student rankings could be made from the 55 entrants?

${}_{55}P_{10} = 1.061374991 \times 10^{17}$

Binomial Distribution

1. Find $P(5)$ if $n = 12$ and $p = .40$ (show setup)

$\binom{12}{5} (.40)^5 (.60)^7 = .227$

2. Find $P(X < 7)$ if $n = 20$ and $p = .85$

$\text{binomcdf}(20, .85, 6) = .0000000459$

3. Find $P(X \geq 9)$ if $n = 25$ and $p = .35$

$1 - \text{binomcdf}(25, .35, 8) = .5332$

4. Find $P(3 < X < 6)$ if $n = 10$ and $p = .42$

$\text{binomcdf}(10, .42, 5) - \text{binomcdf}(10, .42, 3) = .4649$

5. Find $P(3 \leq X \leq 6)$ if $n = 10$ and $p = .42$

$\text{binomcdf}(10, .42, 6) - \text{binomcdf}(10, .42, 2) = .7917$

6. If a dark-haired mother and father have a particular combination of genes, each of their babies have a $\frac{1}{4}$ probability of having light hair.
- What is the probability of any one baby having dark hair? $\frac{3}{4}$
 - If they have 3 babies, calculate $P(0)$, $P(1)$, $P(2)$, and $P(3)$, the probabilities of having 0, 1, 2, and 3 dark-haired babies.
 - Plot the graph of this probability distribution. Does this distribution have a special name?

b) $P(0) = \text{binompdf}(3, \frac{3}{4}, 0) = .0156$

$P(1) = .1406$

$P(2) = .4219$

$P(3) = .4219$



7. Statistics show that about 8% of all males are color-blind. Supposed 20 males are selected at random.
- Find the probability that 2 of the males are color-blind. $\text{binompdf}(20, .08, 2) = .2711$
 - Find the probability that more than 5 of the males are color-blind. $.0038$
 - Find the probability that less than 3 of the males are color-blind. $.7879$
 - Find the probability that between 4 and 7 (inclusive) of the males are color-blind. $.0706$
8. Large tractor-trailer trucks usually have 18 tires. Suppose the probability that any one tire will blow out on a given cross-country trip is 0.03.
- Find the probability that none of the 18 tires blow out. $P(0) = .5780$
 - Find the probability that 1 of the tires blows out. $.3217$
 - Find that probability that more than 2 of the tires blow out. $.0846$
 - If a trucker wants to have a 95% probability of making the trip without a blowout, what must be the reliability of each tire?

$P(0) = .95$

$P(0) = \binom{18}{0} (p)^0 (1-p)^{18}$

$.95 = 1 \cdot 1 \cdot (1-p)^{18}$

$\sqrt[18]{.95} = 1-p$

$.99715 = 1-p$

$p = .0028$