

Prove each identity:

1. $\sec x - \tan x \sin x = \frac{1}{\sec x}$

$$\frac{1}{\cos x} - \frac{\sin x \cdot \sin x}{\cos x}$$

$$\frac{1 - \sin^2 x}{\cos x}$$

$$\frac{\cancel{\cos x}}{\cancel{\cos x}} \rightarrow \cos x \rightarrow \frac{1}{\sec x} \checkmark$$

2. $\frac{1 + \cos x}{\sin x} = \csc x + \cot x$

$$= \frac{1}{\sin x} + \frac{\cos x}{\sin x}$$

$$= \frac{1 + \cos x}{\sin x}$$

3. $\frac{\sec \theta \sin \theta}{\tan \theta + \cot \theta} = \sin^2 \theta$

$$\frac{\frac{1}{\cos \theta} \cdot \frac{\sin \theta}{1}}{\frac{\sin \theta}{\sin \theta} \cdot \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \cdot \frac{\cos \theta}{\cos \theta}} \rightarrow \frac{\sin^2 \theta \cos \theta}{\cos \theta} = \sin^2 \theta$$

$$\frac{\sin \theta}{\cos \theta} \cdot \frac{\sin \theta \cos \theta}{1}$$

4. $\frac{\sec \theta}{\cos \theta} - \frac{\tan \theta}{\cot \theta} = 1$

$$\frac{\frac{1}{\cos \theta}}{\cos \theta} - \frac{\frac{\sin \theta}{\cos \theta}}{\frac{\cos \theta}{\sin \theta}} \rightarrow \frac{1 - \sin^2 \theta}{\cos^2 \theta}$$

$$\frac{\frac{1}{\cos \theta} \cdot \frac{1}{\cos \theta} - \frac{\sin \theta}{\cos \theta} \cdot \frac{\sin \theta}{\cos \theta}}{\frac{1}{\cos^2 \theta} - \frac{\sin^2 \theta}{\cos^2 \theta}} \rightarrow \frac{\cos^2 \theta}{\cos^2 \theta} = 1 \checkmark$$

5. $\cos^2 y - \sin^2 y = 1 - 2\sin^2 y$

$$1 - \sin^2 y - \sin^2 y$$

$$1 - 2\sin^2 y \checkmark$$

6. $\csc^2 \theta \tan^2 \theta - 1 = \tan^2 \theta$

$$\frac{1}{\sin^2 \theta} \cdot \frac{\sin^2 \theta}{\cos^2 \theta} - 1 \rightarrow \frac{\sin^2 \theta}{\cos^2 \theta}$$

$$\frac{1}{\cos^2 \theta} - 1 \cdot \frac{\cos^2 \theta}{\cos^2 \theta}$$

$$\frac{1 - \cos^2 \theta}{\cos^2 \theta} \rightarrow \tan^2 \theta \checkmark$$

7. $\frac{\sec^2 \theta}{\sec^2 \theta - 1} = \csc^2 \theta$

$$\frac{\sec^2 \theta}{\tan^2 \theta}$$

$$\frac{\frac{1}{\cos^2 \theta}}{\frac{\sin^2 \theta}{\cos^2 \theta}} \rightarrow \frac{1}{\sin^2 \theta}$$

$$\frac{1}{\cos^2 \theta} \cdot \frac{\cos^2 \theta}{\sin^2 \theta} \rightarrow \csc^2 \theta \checkmark$$

8. $\tan^2 x \sin^2 x = \tan^2 x - \sin^2 x$

$$= \frac{\sin^2 \theta}{\cos^2 \theta} - \frac{\sin^2 \theta \cdot \cos^2 \theta}{\cos^2 \theta}$$

$$= \frac{\sin^2 \theta (1 - \cos^2 \theta)}{\cos^2 \theta}$$

$$= \tan^2 \theta (1 - \cos^2 \theta)$$

$$= \tan^2 \theta \cdot \sin^2 \theta \checkmark$$

$$9. (\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = 2$$

$$\underbrace{\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta}_{1} + \underbrace{\sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta}_{1} = 2 \checkmark$$

$$10. (\sin\theta + \cos\theta)(\tan\theta + \cot\theta) = \sec\theta + \csc\theta$$

$$\sin\theta\tan\theta + \sin\theta\cot\theta + \cos\theta\tan\theta + \cos\theta\cot\theta$$

$$\sin\theta \cdot \frac{\sin\theta}{\cos\theta} + \sin\theta \frac{\cos\theta}{\sin\theta} + \cos\theta \cdot \frac{\sin\theta}{\cos\theta} + \cos\theta \frac{\cos\theta}{\sin\theta}$$

$$\frac{\sin^2\theta}{\cos\theta} + \cos\theta \frac{\cos\theta}{\cos\theta} + \sin\theta \frac{\sin\theta}{\sin\theta} + \frac{\cos^2\theta}{\sin\theta}$$

$$\frac{\sin^2\theta + \cos^2\theta}{\cos\theta} + \frac{\sin^2\theta + \cos^2\theta}{\sin\theta} \rightarrow \frac{1}{\cos\theta} + \frac{1}{\sin\theta} \checkmark$$

$$11. \frac{\tan\theta - 1}{\tan\theta + 1} = \frac{1 - \cot\theta}{1 + \cot\theta}$$

$$\frac{\frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\cos\theta}}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta}} = \frac{\frac{\sin\theta - \cos\theta}{\cos\theta} \cdot \frac{\cos\theta}{\sin\theta + \cos\theta}}{\frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta}}$$

$$\frac{\sin\theta - \cos\theta}{\cos\theta} \rightarrow \frac{1 - \cot\theta}{1 + \cot\theta} \checkmark$$

$$12. \frac{1 - \tan^2 x}{1 + \tan^2 x} = 1 - 2\sin^2 x$$

$$\frac{1 - \frac{\sin^2 x}{\cos^2 x}}{\frac{\cos^2 x - \sin^2 x}{\cos^2 x}} = \frac{1 - \sin^2 x - \sin^2 x}{1 - 2\sin^2 x} \checkmark$$

$$\frac{\cos^2 x - \sin^2 x}{\cos^2 x} \cdot \frac{\cos^2 x}{1} = \frac{\cos^2 x - \sin^2 x}{\cos^2 x - \sin^2 x} \checkmark$$

$$13. \frac{\cos x + 1}{\sin^3 x} = \frac{\csc x}{1 - \cos x} \cdot \frac{1 + \cos x}{1 + \cos x}$$

$$= \frac{\csc x + \csc x \cos x}{1 - \cos^2 x}$$

$$= \frac{\frac{1}{\sin x} + \frac{1}{\sin x} \cdot \cos x}{\sin^2 x}$$

$$= \frac{1 + \cos x}{\sin x} \cdot \frac{1}{\sin^2 x} = \frac{1 + \cos x}{\sin^3 x} \checkmark$$

$$14. \csc^4 x - \cot^4 x = \csc^2 x + \cot^2 x$$

$$(\csc^2 x + \cot^2 x)(\csc^2 x - \cot^2 x)$$

$$\csc^2 x + \cot^2 x \checkmark$$

$$15. \frac{\tan\theta}{\sec\theta} + \frac{\cot\theta}{\csc\theta} = \sin\theta + \cos\theta$$

$$\frac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}}$$

$$\frac{\sin\theta \cdot \cos\theta}{\cos\theta} + \frac{\cos\theta \cdot \sin\theta}{\sin\theta}$$

$$\sin\theta + \cos\theta \checkmark$$

$$16. \frac{\sin y + \tan y}{1 + \sec y} = \sin y$$

$$\frac{\frac{\sin y \cos y}{\cos y} + \frac{\sin y}{\cos y}}{\frac{\cos y}{\cos y} + \frac{1}{\cos y}}$$

$$\frac{\sin y (1 + \cos y)}{\cos y} \cdot \frac{\cos y}{1 + \cos y} = \sin y \checkmark$$