Unit 4 Review

1. $\sin(97° + 43°) = \sin 140°$

2. $\cos(72 - 130) = \cos(-58)$

3. $\tan\left(\frac{\pi}{3} + \frac{\pi}{4}\right) = \tan\left(\frac{7\pi}{12}\right)$

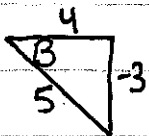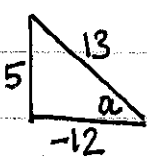4. $\sin\left(\frac{\pi}{5} - \frac{2\pi}{3}\right) = \sin\left(\frac{-7\pi}{15}\right)$

   * Same as $-\sin\left(\frac{7\pi}{15}\right)$
   * identity

5. $\tan(45° - 150°) = \dfrac{\tan 45° - \tan 150°}{1 + \tan 45° \cdot \tan 150°} = \dfrac{1 - \left(-\frac{\sqrt{3}}{3}\right)}{1 + \left(-\frac{\sqrt{3}}{3}\right)} = \dfrac{\frac{3+\sqrt{3}}{3}}{\frac{3-\sqrt{3}}{3}} = \dfrac{3+\sqrt{3}}{3-\sqrt{3}} \cdot \dfrac{(3+\sqrt{3})}{(3+\sqrt{3})}$

   $= \dfrac{9 + 6\sqrt{3} + 3}{9 - 3}$

   $= \dfrac{12 + 6\sqrt{3}}{6} = 2 + \sqrt{3}$

6. $\sin(300° + 45°) = \sin 300° \cos 45° + \cos 300° \sin 45° = \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

   $= -\dfrac{\sqrt{6}}{4} + \dfrac{\sqrt{2}}{4} = \dfrac{-\sqrt{6} + \sqrt{2}}{4}$

7. $\sin\alpha\cos\beta - \cos\alpha\sin\beta$

   $\left(\frac{5}{13}\right)\left(\frac{4}{5}\right) - \left(\frac{-12}{13}\right)\left(\frac{-3}{5}\right)$

   $= \dfrac{20}{65} - \dfrac{36}{65} = \dfrac{-16}{65}$

8. $\cos\beta\cos\alpha - \sin\beta\sin\alpha$

   $\left(\frac{4}{5}\right)\left(\frac{-12}{13}\right) - \left(\frac{-3}{5}\right)\left(\frac{5}{13}\right)$

   $= \dfrac{-48}{65} + \dfrac{15}{65} = \dfrac{-33}{65}$

9. $\dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

   $= \dfrac{\left(\frac{5}{-12}\right) - \left(\frac{-3}{4}\right)}{1 + \left(\frac{5}{12}\right)\left(\frac{-3}{4}\right)}$

   $= \dfrac{\frac{4}{12}}{1 + \frac{15}{48}} = \dfrac{\frac{1}{3}}{\frac{63}{48}} = \dfrac{48}{189}$

   $= \dfrac{16}{63}$

10. $\cos\theta\cos\frac{\pi}{3} - \sin\theta\sin\frac{\pi}{3}$

$$= \left(-\frac{4}{5}\right)\left(\frac{1}{2}\right) - \left(-\frac{3}{5}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{-4}{10} + \frac{3\sqrt{3}}{10} = \boxed{\frac{-4+3\sqrt{3}}{10}}$$

11. $\dfrac{2\tan\theta}{1-\tan^2\theta} = \dfrac{2\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)^2} = \dfrac{\frac{3}{2}}{1-\frac{9}{16}} = \dfrac{\frac{3}{2}}{\frac{7}{16}}$

$$= \frac{3}{2} \cdot \frac{16}{7} = \boxed{\frac{24}{7}}$$

12. Messing with left side:

$$\frac{(\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)}{(1-\sin^2 x - \sin^2 x)\;(1)}$$

$$1 - 2\sin^2 x \checkmark$$

13. Messing with left side:

$$\frac{(\sec x +1)\cos x}{(\sec x+1)(\sec x-1)} - \frac{\cos x}{\tan^2 x} =$$

$$\frac{\left(\frac{1}{\cos x} + \frac{\cos x}{\cos x}\right)\cos x - \cos x}{\tan^2 x} =$$

$$\frac{1 + \cos x - \cos x}{\tan^2 x} = \frac{1}{\tan^2 x} = \cot^2 x \checkmark$$

14. $\sin\pi\cos x - \cos\pi\sin x =$

$$0 - (-1)\sin x$$

$$\sin x = \checkmark$$

15. $\sin\frac{3\pi}{2}\cos x + \cos\frac{3\pi}{2}\sin x$

$$-\cos x + 0\sin x =$$

$$-\cos x \checkmark$$

16. $\cos\alpha\cos\beta - \sin\alpha\sin\beta + \cos\alpha\cos\beta + \sin\alpha\cos\beta =$

$$2\cos\alpha\cos\beta = \checkmark$$

17. $\dfrac{\sin\beta\cos\alpha - \cos\beta\sin\alpha}{\sin\alpha\sin\beta} =$

$$\frac{\cos\alpha}{\sin\alpha} - \frac{\cos\beta}{\sin\beta} =$$

$$\cot\alpha - \cot\beta = \checkmark$$

18. $\sin x \cos \frac{\pi}{6} + \cos x \sin \frac{\pi}{6} - \left(\sin x \cos \frac{\pi}{6} - \cos x \sin \frac{\pi}{6}\right) = \frac{1}{2}$

$\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x = \frac{1}{2}$

$\cos x = \frac{1}{2}$ $X = \frac{\pi}{3}$ and $\frac{5\pi}{3}$

19. $\dfrac{\tan x + \tan \pi}{1 - \tan x \tan \pi} + 2\left(\sin x \cos \pi + \cos x \sin \pi\right) = 0$

$\frac{\tan x}{1} + -2\sin x = 0 \qquad\longrightarrow\qquad \sin x\left(\frac{1}{\cos x} - 2\right) = 0$

$\frac{\sin x}{\cos x} - 2\sin x = 0$

$\sin x = 0$ → $X = 0, \pi$

$\frac{1}{\cos x} - 2 = 0$

$\frac{1}{\cos x} = 2$

$\cos x = \frac{1}{2}$

$X = \frac{\pi}{3}$ and $\frac{5\pi}{3}$

20. $\sin x \cos \frac{\pi}{2} + \cos x \sin \frac{\pi}{2} - \left(\cos x \cos \frac{3\pi}{2} - \sin x \sin \frac{3\pi}{2}\right) = 0$

$0 + \cos x - 0 + (-1)\sin x = 0$

$\cos x - \sin x = 0$

$\cos x = \sin x$

$X = \frac{\pi}{4}$ and $\frac{5\pi}{4}$