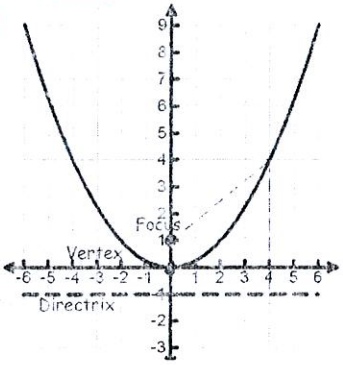
Notes: Parabolas

Parabola: the set of all points equidistant from a point called the Focus and a line called the directrix

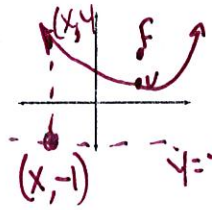
The distance between the focus and a point on the curve = the distance between the directrix and a point on the curve.



The Focus is the focus because...

http://www.ies.co.jp/math/java/conics/focus/focus.html

Ok... so using the definition, let's come up with the equation of a parabola:



Given:

Focus (2,3)

Vertex (2,1)

distance $(x,y) \to$ Focus must be same as distance $(x,y) \to$ directrix

$$\sqrt{(x-2)^2+(y-3)^2} = \sqrt{(x-x)^2+(y+1)^2}$$

$$x^2-4x+4+y^2-6y+9 = 0+y^2+2y+1$$

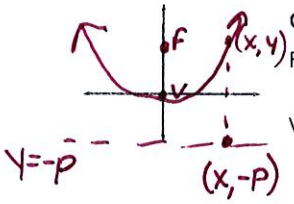
$$x^2-4x+13-1 = 8y$$

$$y = \frac{1}{8}x^2 - \frac{1}{2}x + \frac{3}{2}$$

Quadratic from Alg I with

$a = \frac{1}{8}$ $b = \frac{-1}{2}$ $c = \frac{3}{2}$

Now for ANY parabola:



Given:

Focus (0, p)

Vertex (0, 0)

$$\sqrt{(0-x)^2+(y-p)^2} = \sqrt{(x-x)^2+(y+p)^2}$$

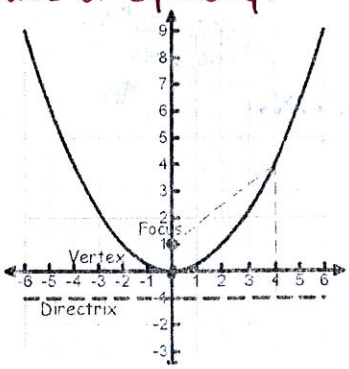
$$x^2+y^2-2py+p^2 = 0+y^2+2py+p^2$$

$$x^2 = 4py$$

Parabola that opens up where p = distance btw vertex and Focus.

The line perpendicular to the parabola's directrix and passing through its Focus and Vertex is the axis of symmetry.

In the parabola to the right, the AoS is the y-axis.



Notes: Parabolas

Conic Form of a parabola opening vertically

$$\boxed{(x-h)^2 = 4p(y-k)}$$

Conic Form of a parabola opening horizontally

$$\boxed{(y-k)^2 = 4p(x-h)}$$

*Hint:
if y-quantity is not squared then opens up & down;
if x-quantity is not squared, then opens left & right.
*note: this is consistent w/ the direction of the x and y axes

$(x-h)^2 = 4p(y-k)$ $\qquad$ $(y-k)^2 = 4p(x-h)$

vertex = (h, k)

p = the distance from the vertex to the focus which is the same as the distance from the vertex to the directrix

*the opening of the parabola is based on the sign of 4p

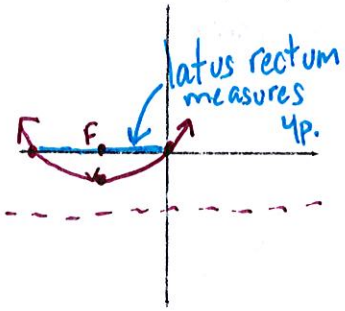
positive 4p: opens up/right

negative 4p: opens down/left

* The "larger" p is, the fatter the graph will be.

EX: For each parabola, write the equation in SF, state the vertex, focus, and directrix and then graph.

a) $(x+2)^2 = 4(y+1)$



1) Is it horizontal or vertical? vertical. up

2) Find the vertex. (-2, -1)

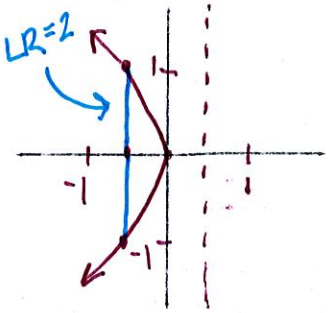
3) Find p. +1

4) Add p to the vertex to find the focus. (-2, 0)

5) Subtract p to find the directrix. y = -2

* add/sub to y if it's vertical.
*add/sub to x if it's horizontal.

b) $y^2 = -2x$



1) Is it horizontal or vertical? Hor. Left.

2) Find the vertex. (0,0)

3) Find p. $-\frac{1}{2}$

4) Add p to the vertex to find the focus. $(-\frac{1}{2}, 0)$

5) Subtract p to find the directrix. $x = \frac{1}{2}$

* add/sub to y if it's vertical.
*add/sub to x if it's horizontal.

Notes: Parabolas

1) Is it horizontal or vertical? Hor. Right

2) Find the vertex. (-3,6)

3) Find p. 4

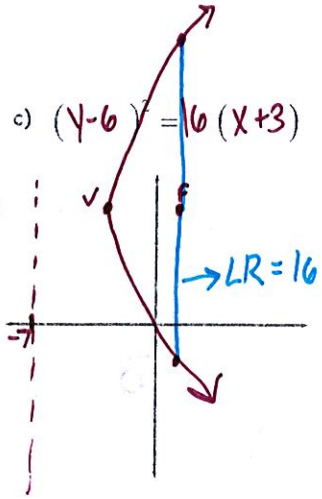
4) Add p to the vertex to find the focus. (1, 6)

5) Subtract p to find the directrix. X = -7

* add/sub to y if it's vertical.

*add/sub to x if it's horizontal.

c) $(y-6)^2 = 16(x+3)$



d) $x^2 = -36(y-1)$

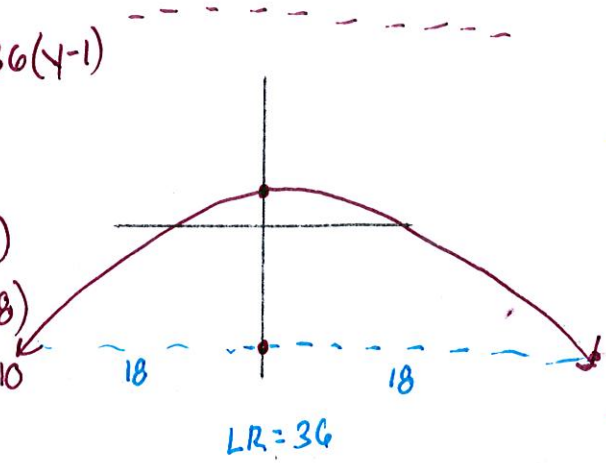
down

$p = -9$

$V = (0,1)$

$F = (0,-8)$

dir: $y = 10$

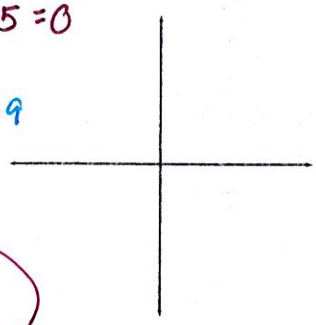


e) $x^2 + 6x - 4y + 5 = 0$

$x^2 + 6x + 9 = 4y - 5 + 9$

$(x+3)^2 = 4y + 4$

$(x+3)^2 = 4(y+1)$



V: (-3, -1)

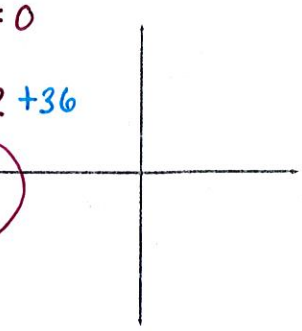
F: (-3,0)

dir: $y = -2$

f) $y^2 - 12y - 16x - 12 = 0$

$y^2 - 12y + 36 = 16x + 12 + 36$

$(y-6)^2 = 16(x+3)$



V: (-3, 6)

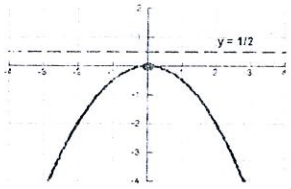
F: (1, 6)

dir: $x = -7$

Notes: Parabolas

Ex: Write an equation of the parabola to the right.

***Which way does the parabola open?
So which standard form are we using?
What do we need to know and how do we find it?

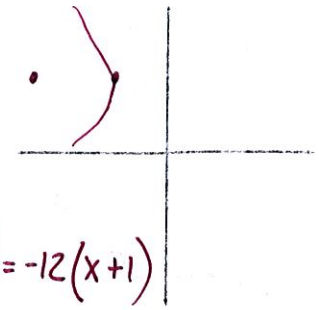


$p = -\frac{1}{2}$

$x^2 = -2y$

Ex: Write the equation of the parabola with vertex $(-1, 3)$ and focus $(-4, 3)$.

*** Which way does the parabola open?
So which standard form are we using?
What do we need to know and how do we find it?

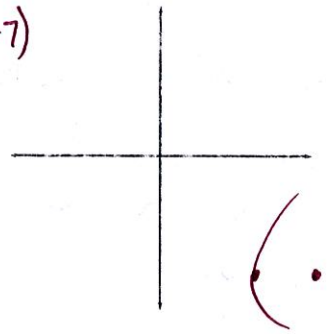


$(y-3)^2 = -12(x+1)$

Ex: Write the equation from the information given:

a. vertex $(4, -7)$, focus $(6, -7)$

$(y+7)^2 = 8(x-4)$



b) vertex $(-4, -2)$ directrix $y = -4$

$(x+4)^2 = 8(y+2)$