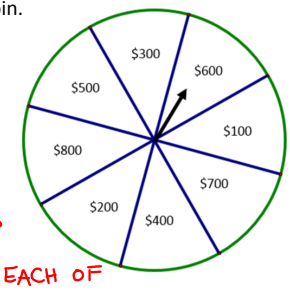
Lesson 2 - Intro to Probability Marked

Section #1 – Basic Probability

Assume that the spinner to the right is fair. Find each of the following probabilities given one spin.



1. P($800) = $\frac{1}{8}$

2. P($400) = $\frac{1}{8}$

3. Does P($100) = P($800)? YES

Explain: THE AREAS OF EACH OF THE 8 SECTORS ARE $\cong$.
$\therefore$ EACH SPACE HAS AN EQUAL CHANCE OF BEING LANDED ON WHEN THE SPINNER IS SPUN.

4. P(at least $500) =

P(500) OR P(600) OR P(700) OR P(800)

$= \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{4}{8}$

5. P(less than $200) =

$P(100) = \frac{1}{8}$

6. P(at most $500) =

P(500) OR P(400) OR P(300) OR P(200) OR P(100)

$= \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{5}{8}$

Calculate the following probabilities given two spins

7. P(sum of $200)

SAMPLE SPACE: (100,100)

P(100 AND 100)

$= \frac{1}{8} \cdot \frac{1}{8}$

$= \frac{1}{64}$

8. P(sum of at most $400)

POSSIBLE SUMS	SAMPLE SPACE		
400	(300,100)	(200,200)	(100,300)
300	(200,100)	(100,200)	
200	(100,100)		

P(sum=400) OR P(sum=300) OR P(sum=200)

P(300,100) OR P(200,200) OR P(100,300) OR P(200,100) OR P(100,200) OR P(100,100)

$= (\frac{1}{8}\cdot\frac{1}{8}) + (\frac{1}{8}\cdot\frac{1}{8}) + (\frac{1}{8}\cdot\frac{1}{8}) + (\frac{1}{8}\cdot\frac{1}{8}) + (\frac{1}{8}\cdot\frac{1}{8}) + (\frac{1}{8}\cdot\frac{1}{8})$

$= \frac{1}{64} + \frac{1}{64} + \frac{1}{64} + \frac{1}{64} + \frac{1}{64} + \frac{1}{64}$

$= \frac{6}{64}$

9. P(sum of at least $1500)

POSSIBLE SUMS	SAMPLE SPACE	
1500	(700,800)	(800,700)
1600	(800,800)	

P(sum=1500) OR P(sum=1600)

= P(700,800) OR P(800,700) OR P(800,800)

$= (\frac{1}{8}\cdot\frac{1}{8}) + (\frac{1}{8}\cdot\frac{1}{8}) + (\frac{1}{8}\cdot\frac{1}{8})$

$= \frac{1}{64} + \frac{1}{64} + \frac{1}{64}$

$= \frac{3}{64}$

10. P(sum of at least $300)

* TOO MANY SAMPLE SPACE ITEMS!

P(EVENT) + P(COMPLEMENT OF EVENT) = 1

$\Rightarrow$ P(EVENT) = 1 − P(COMPLEMENT OF EVENT)

EVENT: AT LEAST 300

COMPLEMENT OF EVENT: LESS THAN 300

P(LESS THAN 300)

= P(200) = P(100 AND 100)

$= \frac{1}{8} \cdot \frac{1}{8} = \frac{1}{64}$

$\therefore$ P(AT LEAST 300)

= 1 − P(LESS THAN 300)

$= 1 - \frac{1}{64}$

$= \frac{63}{64}$

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11. P(sum of \$200 | first spin lands on \$100)

"GIVEN THAT"

SAMPLE SPACE: (100, 100)

P(100 ON 1st SPIN) = 1

P(100 ON 2ND SPIN) = 1/8

∴ P(100 AND 100)

= 1 · 1/8

= 1/8

12. P(sum of at least \$1000 | 1st spin lands on 800)

= 1 − P(SUM IS LESS THAN 1000 | 1st SPIN IS 800)

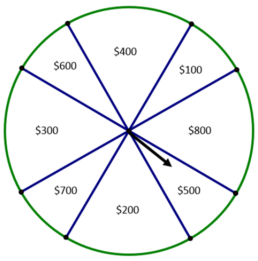
= 1 − P(800, 100)

= 1 − (1 · 1/8)

= 1 − 1/8

= 7/8

In the new spinner to the right, assume that the \$100, \$500, \$700 and \$600 regions are half the size of the \$800, \$200, \$300 and \$400 regions.



13. P(\$800) = 2/12

14. P(\$400) = 2/12

15. Does P(\$100) = P(\$800)? NO

Explain: THE AREAS OF EACH OF THE 8 SECTORS ARE NOT ≅.

∴ EACH SPACE DOES NOT HAVE EQUAL CHANCE OF BEING LANDED ON WHEN THE SPINNER IS SPUN.

16. P(at least \$500) =

P(500) OR P(600) OR P(700) OR P(800)

= 1/12 + 1/12 + 1/12 + 2/12 = 5/12

17. P(less than \$200) =

P(100) = 1/12

18. P(at most \$500) =

P(500) OR P(400) OR P(300) OR P(200) OR P(100)

= 1/12 + 2/12 + 2/12 + 2/12 + 1/12 = 8/12

Calculate the following probabilities given two spins

19. P(sum of \$200)

SAMPLE SPACE: (100, 100)

P(100 AND 100)

= 1/12 · 1/12

= 1/144

20. P(sum of at most \$400)

POSSIBLE SUMS	SAMPLE SPACE		
400	(300, 100)	(200, 200)	(100, 300)
300	(200, 100)	(100, 200)	
200	(100, 100)		

P(SUM = 400) OR P(SUM = 300) OR P(SUM = 200)

P(300,100) OR P(200,200) OR P(100,300) OR P(200,100) OR P(100,200) OR P(100,100)

= (2/12 · 1/12) + (2/12 · 2/12) + (1/12 · 2/12) + (2/12 · 1/12) + (1/12 · 2/12) + (1/12 · 1/12)

= 2/144 + 4/144 + 2/144 + 2/144 + 2/144 + 1/144

= 13/144

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21. P(sum of at least $1500)

POSSIBLE SUMS	SAMPLE SPACE	
1500	(700,800)	(800,700)
1600	(800,800)	

P(sum = 1500) OR P(sum = 1600)

= P(700,800) OR P(800,700) OR P(800,800)

$= \left(\frac{1}{12}\cdot\frac{2}{12}\right)+\left(\frac{2}{12}\cdot\frac{1}{12}\right)+\left(\frac{2}{12}\cdot\frac{2}{12}\right)$

$= 2/144 + 2/144 + 4/144$

$= 8/144$

22. P(sum of at least $300)

* TOO MANY SAMPLE SPACE ITEMS!

P(EVENT) + P(COMPLEMENT OF EVENT) = 1

⇒ P(EVENT) = 1 − P(COMPLEMENT OF EVENT)

EVENT: AT LEAST 300

COMPLEMENT OF EVENT: LESS THAN 300

P(LESS THAN 300)

= P(200) = P(100 AND 100)

$= 1/12 \cdot 1/12 = 1/144$

∴ P(AT LEAST 300)

= 1 − P(LESS THAN 300)

$= 1 - 1/144$

$= 143/144$

23. P(sum of $200 | first spin lands on $100)

"GIVEN THAT"

SAMPLE SPACE: (100, 100)

P(100 ON 1ST SPIN) = 1

P(100 ON 2ND SPIN) = 1/12

∴ P(100 AND 100)

$= 1 \cdot 1/12$

$= 1/12$

24. P(sum of at least $1000 | 1st spin lands on 800)

= 1 − P(SUM IS LESS THAN 1000 | 1ST SPIN IS 800)

= 1 − P(800, 100)

$= 1 - (1 \cdot 1/12)$

$= 1 - 1/12$

$= 11/12$

In this last spinner, we are not given any information about the areas of each section.

25. How could we determine the probability of landing on each individual amount?

26. Determine each of the following probabilities:

P($100) = 60/360

P($200) = 75/360

P($300) = 55/360

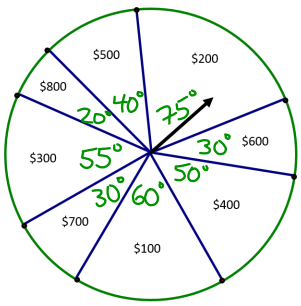
P($400) = 50/360

P($500) = 40/360

P($600) = 30/360

P($700) = 30/360

P($800) = 20/360



Calculate the following based on one spin:

27. P(at least $500)

P(500) OR P(600) OR P(700) OR P(800)

$= \frac{40}{360} + \frac{30}{360} + \frac{30}{360} + \frac{20}{360}$

$= 120/360$

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28. P(less than $200)

$P(100) = 60/360$

29. P(at most $500)

$P(500)$ OR $P(400)$ OR $P(300)$ OR $P(200)$ OR $P(100)$

$= \frac{40}{360} + \frac{50}{360} + \frac{55}{360} + \frac{75}{360} + \frac{60}{360}$

$= 280/360$

Calculate the following based on two spins:

30. P(a sum of $200)

$P(100,100)$

$= \frac{60}{360} \cdot \frac{60}{360}$

$= 1/6 \cdot 1/6$

$= 1/36$

31. P(a sum of at most $300)

POSSIBLE SUMS	SAMPLE SPACE	
300	(200,100)	(100,200)
200	(100,100)	

$P(200,100) + P(100,200) + P(100,100)$

$= \left(\frac{75}{360} \cdot \frac{60}{360}\right) + \left(\frac{60}{360} \cdot \frac{75}{360}\right) + \left(\frac{60}{360} \cdot \frac{60}{360}\right)$

$= \frac{4500}{129600} + \frac{4500}{129600} + \frac{3600}{129600}$

$= \frac{12600}{129600}$

32. P(at least $1500)

POSSIBLE SUMS	SAMPLE SPACE	
1500	(700,800)	(800,700)
1600	(800,800)	

$P(700,800) + P(800,700) + P(800,800)$

$= \left(\frac{30}{360} \cdot \frac{20}{360}\right) + \left(\frac{20}{360} \cdot \frac{30}{360}\right) + \left(\frac{20}{360} \cdot \frac{20}{360}\right)$

$= \frac{600}{129600} + \frac{600}{129600} + \frac{400}{129600}$

$= \frac{1600}{129600}$

Section #2 – Probability of Overlapping Events

A card is randomly chosen from a standard deck of 52 playing cards.

33. Find the probability that the card is a 9 <u>or</u> a king.

INTERSECT

$P(9) + P(K) - P(9 \cap K)$

$= \frac{4}{52} + \frac{4}{52} - \frac{0}{52}$

$= \frac{8}{52}$

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34. Find the probability that the card is an ace or a spade.

$P(\text{ACE}) + P(\text{SPADE}) - P(A \cap Sp)$

$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$

$= 16/52$

35. Find the probability that the card is the King of Diamonds or any spade.

$P(KD) + P(Sp) - P(KD \cap Sp)$

$= \frac{1}{52} + \frac{13}{52} - \frac{0}{52}$

$= 14/52$

36. Find the probability that the card is red or a face card.

$P(\text{RED}) + P(\text{FACE}) - P(\text{RED} \cap \text{FACE})$

$= \frac{26}{52} + \frac{12}{52} - \frac{6}{52}$

$= 32/52$

You draw a card from a bag that contains 4 yellow cards numbered 1-4 and 4 blue cards numbered 1-4. Tell whether the events A and B are mutually exclusive or overlapping. Then find P(A or B).

37. Event A: you choose a yellow card
Event B: you choose a blue card

$P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$

$P(Y) + P(B) - P(Y \cap B)$

$= \frac{4}{8} + \frac{4}{8} - \frac{0}{8}$

$= 1$

MUTUALLY EXCLUSIVE

38. Event A: you choose a blue card
Event B: you choose a #3 card

$P(\text{BLUE}) + P(\#3) - P(\text{BLUE} \cap \#3)$

$= \frac{4}{8} + \frac{2}{8} - \frac{1}{8}$

$= 5/8$

OVERLAPPING

39. Event A: you choose a #1 card
Event B: you choose a yellow card

$P(\#1) + P(\text{YELLOW}) - P(\#1 \cap \text{YELLOW})$

$= \frac{2}{8} + \frac{4}{8} - \frac{1}{8}$

$= 5/8$

OVERLAPPING

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40. Event A: you choose an odd-numbered card
Event B: you choose a #2 card

$P(\text{ODD}) + P(\#2) - P(\text{ODD} \cap \#2)$

$= \frac{4}{8} + \frac{2}{8} - \frac{0}{8}$

$= 6/8$

MUTUALLY EXCLUSIVE

41. Event A: you choose a blue #4 card
Event B: you choose a blue card

$P(\text{BLUE } \#4) + P(\text{BLUE}) - P(\text{BLUE } \#4 \cap \text{BLUE})$

$= \frac{1}{8} + \frac{4}{8} - \frac{1}{8}$

$= 4/8$

OVERLAPPING

42. Event A: you choose an even-numbered card
Event B: you choose a yellow card

$P(\text{EVEN}) + P(\text{YELLOW}) - P(\text{EVEN} \cap \text{YELLOW})$

$= \frac{4}{8} + \frac{4}{8} - \frac{2}{8}$

$= 6/8$

OVERLAPPING

<u>Section #3 – Dependent vs Independent Events</u>

There are 21 marbles in a bag. Seven are blue, seven are red, and seven are green. If a blue marble is drawn from the bag and not replaced, calculate the probability of each of the following occurring on the 2nd draw.

43. P(blue) = $6/20$

44. P(blue or green) =

$P(\text{BLUE}) + P(\text{GREEN})$

$= \frac{6}{20} + \frac{7}{20}$

$= 13/20$

45. P(not blue) =

$P(\text{RED OR GREEN})$

$= P(\text{RED}) + P(\text{GREEN})$

$= \frac{7}{20} + \frac{7}{20}$

$= 14/20$

Lesson 2 - Intro to Probability Marked

There are 21 marbles in a bag. Seven are blue, seven are red, and seven are green. If the marbles are not replaced after the 1st draw, what is the probability of each of the following occurring in the given order?

46. P(red, blue)

$= P(\text{RED}) \text{ AND } P(\text{BLUE})$

$= \frac{7}{21} \cdot \frac{7}{20}$

$= \frac{1}{3} \cdot \frac{7}{20}$

$= 7/60$

47. P(red, blue, green)

$P(\text{RED}) \text{ AND } P(\text{BLUE}) \text{ AND } P(\text{GREEN})$

$= \frac{7}{21} \cdot \frac{7}{20} \cdot \frac{7}{19}$

$= \frac{1}{3} \cdot \frac{7}{20} \cdot \frac{7}{19}$

$= \frac{49}{1140}$

48. P(red or blue, green)

$= [P(\text{RED}) \text{ OR } P(\text{BLUE})] \cdot P(\text{GREEN})$

$= \left[\frac{7}{21} + \frac{7}{21}\right] \cdot \frac{7}{20}$

$= \frac{14}{21} \cdot \frac{7}{20}$

$= \frac{2}{3} \cdot \frac{7}{20}$

$= \frac{1}{3} \cdot \frac{7}{10}$

$= 7/30$

49. P(red | first draw was a red marble)

6 REDS
20 TOTAL

$6/20$

Using a standard deck of 52 cards, 3 cards are dealt without replacement.

50. What is the probability that all three cards are queens?

$\frac{4}{52} \cdot \frac{3}{51} \cdot \frac{2}{50}$

$= \frac{1}{13} \cdot \frac{1}{17} \cdot \frac{1}{25}$

$= 1/5525$

51. What is the probability of being dealt three of a kind?

$\frac{4}{52} \cdot \frac{3}{51} \cdot \frac{2}{50} \cdot 13$

$= \frac{3}{51} \cdot \frac{2}{50}$

$= \frac{1}{17} \cdot \frac{1}{25}$

$= 1/425$

OR

$\frac{52}{52} \cdot \frac{3}{51} \cdot \frac{2}{50}$

$= 1 \cdot \frac{1}{17} \cdot \frac{1}{25}$

$= 1/425$

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52. If the first card is an Ace, what is the probability that the second card will not be an Ace?

3 ACES LEFT

51 CARDS LEFT

P(NOT AN ACE) = 48/51

53. If the first two cards are queens, what is the probability that you will be dealt three queens?

2 QUEENS LEFT

50 CARDS LEFT

P(3 TOTAL QUEENS) = 2/50

54. What is the probability of being dealt three cards from the same suit?

$$\frac{13}{52} \cdot \frac{12}{51} \cdot \frac{11}{50} \cdot 4$$

$$= \frac{1}{4} \cdot \frac{4}{17} \cdot \frac{11}{50} \cdot 4$$

$$= \frac{4}{17} \cdot \frac{11}{50}$$

$$= \frac{2}{17} \cdot \frac{11}{25}$$

$$= 22/425$$

OR

$$\frac{52}{52} \cdot \frac{12}{51} \cdot \frac{11}{50}$$

$$= 1 \cdot \frac{4}{17} \cdot \frac{11}{50}$$

$$= \frac{2}{17} \cdot \frac{11}{25}$$

$$= 22/425$$

55. Find the probability of getting a run of 3, 4, 5, all of the same suit, but not necessarily in numerical order

$$\frac{12}{52} \cdot \frac{2}{51} \cdot \frac{1}{50}$$

$$= \frac{3}{13} \cdot \frac{1}{51} \cdot \frac{1}{25}$$

$$= \frac{1}{13} \cdot \frac{1}{17} \cdot \frac{1}{25}$$

$$= 1/5525$$

A student conducted a survey with a randomly selected group of students. She asked freshmen, sophomores, juniors, and seniors to tell her whether or not they liked the school cafeteria food.

	Freshmen	Sophomores	Juniors	Seniors	
Liked food	85	50	77	82	294
Did not like food	44	92	56	78	270
	129	142	133	160	564

56. What is the probability that a randomly selected student is a freshman?

129/564

57. What is the probability that a randomly selected student likes the food?

294/564

58. What is the probability that a randomly selected student is a freshman and likes the food?

85/564

59. If the randomly selected student likes the food, what is the probability that he/she is a freshman?

85/294