Pre-Calculus Midterm Review Part 1 Spring 2019 Name: Answer key

1. Convert 184° to radian measure. $184° \cdot \frac{\pi}{180°} = \boxed{\frac{46\pi}{45}}$

2. Convert $\frac{7\pi}{10}$ to degree measure. $\frac{7\pi}{10} \cdot \frac{180}{\pi} = \boxed{126°}$

3. State the reference angle for 1920°.
$1920° - 360° - 360° \ldots = 120°$ Ref ∠ $= \boxed{60°}$

4. State a positive coterminal angle of $\frac{43\pi}{6}$.
$2\pi = \frac{12\pi}{6}$ $\frac{43\pi}{6} + \frac{12\pi}{6} =$ $\boxed{\frac{55\pi}{6} \text{ or } \frac{31\pi}{6} \text{ or } \frac{7\pi}{6} \text{ or } \frac{19\pi}{6} \ldots}$

5. State a negative coterminal angle of 1020°.
$1020° - 360° - 360° \ldots = \boxed{-60°}$

6. Evaluate each of the following.

a. $\cos^{-1}\left(-\frac{1}{2}\right)$ 120° or $\frac{2\pi}{3}$

b. $\arctan(-1)$ −45° or $\frac{-\pi}{4}$

c. $\sin^{-1}\left(-\frac{1}{2}\right)$ −30° or $\frac{-\pi}{6}$

d. $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ 30° or $\frac{\pi}{6}$

e. $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ 315° or $\frac{7\pi}{4}$

f. $\tan^{-1}\left(\sqrt{3}\right)$ 60° or $\frac{\pi}{3}$

Given the function $h(x) = -7\cos\left(\frac{1}{3}x - \frac{\pi}{4}\right) - 3$, state each of the following characteristics.

7. phase shift Right $\frac{3\pi}{4}$

8. period 6π

9. amplitude 7

10. sinusoidal axis $y = -3$

11. Evaluate. $\cos\left(\underbrace{\sin^{-1}\left(-\frac{1}{2}\right)}_{-30°}\right) = \boxed{\frac{\sqrt{3}}{2}}$

12. Evaluate. $\sin\left(\underbrace{\arctan\left(-\sqrt{3}\right)}_{-60°}\right) = \boxed{-\frac{\sqrt{3}}{2}}$

The diameter of a Ferris wheel is 176 feet, and one complete revolution takes 11 minutes. The bottom of the wheel is 10 feet above the ground. Each passenger gets on the ride at the bottom of the wheel.

13. What is the highest distance above the ground that each passenger will attain during the ride?

186 ft

14. State an equation that will give each rider's height above the ground at any time during the ride.

$y = -88\cos\left(\frac{2\pi}{11}t\right) + 98$

15. The equation $D(t) = 8\cos\left(\frac{\pi}{8}t\right) + 13$ models water depth in meters in a seaport with $t = 0$ representing 6:00 AM.

a. What will the water depth be at 3 PM? $\rightarrow$ 9 hours after 6 AM

$8\cos\left(\frac{\pi}{8}(9)\right) + 13$ $\boxed{5.61\text{ m}}$

b. At what time after 6:00 AM will the water depth first be 8 meters? State your answer in hours and minutes, and be sure to designate AM or PM.

5.72 hours after 6 AM $\rightarrow$ $\boxed{\text{11:43 AM}}$

16. Determine the area of a triangle with side lengths $a = 13$, $b = 16$, and $c = 9$.

$s = 19$ $A = \sqrt{19(6)(3)(10)} = \boxed{58.48}$

17. Determine the area of a triangle given $a = 90$, $b = 28$, and $C = 50^\circ$.

$A = \frac{1}{2}(90)(28)\sin 50^\circ = \boxed{965.22}$

18. Given $A = 121^\circ$, $a = 15$, and $b = 11$, determine the measure of angle B.

$\frac{\sin 121^\circ}{15} = \frac{\sin B}{11}$ $\boxed{\angle B = 38.95^\circ}$

19. Given $a = 14$, $b = 17$, and $c = 7$, determine the measure of angle A.

$14^2 = 17^2 + 7^2 - 2(17)(7)\cos A$

$\boxed{A = 53.37^\circ}$

Pre-Calculus Midterm Review – Part 2 Spring 2019 Name: Answer Key

20. Use difference formulas to determine the exact values of $\sin(15^\circ)$, $\cos(15^\circ)$, and $\tan(15^\circ)$.

$\sin 15^\circ = \sin(150^\circ - 135^\circ) = \sin 150^\circ \cos 135^\circ - \cos 150^\circ \sin 135^\circ = \left(\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) - \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

$\cos 15^\circ = \cos(225^\circ - 210^\circ) = \cos 225^\circ \cos 210^\circ + \sin 225^\circ \sin 210^\circ = \left(-\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{3}}{2}\right) + \left(-\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right) = \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$

$\tan 15^\circ = \tan(45^\circ - 30^\circ) = \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ} = \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} = \frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}} = \boxed{\frac{\sqrt{3}-1}{\sqrt{3}+1}}$ Same as: $\boxed{\frac{3-\sqrt{3}}{3+\sqrt{3}}}$

21. Use a sum formula to determine the exact value of $\tan 165^\circ$.

$\tan(135^\circ + 30^\circ) = \frac{\tan 135^\circ + \tan 30^\circ}{1 - \tan 135^\circ \tan 30^\circ} = \frac{-1 + \frac{1}{\sqrt{3}}}{1 - (-1)\left(\frac{1}{\sqrt{3}}\right)} = \frac{\frac{1-\sqrt{3}}{\sqrt{3}}}{\frac{1+\sqrt{3}}{\sqrt{3}}} = \boxed{\frac{1-\sqrt{3}}{1+\sqrt{3}}}$

22. Justify. $(\tan x)(\csc x)(1 - \sin^2 x) = \frac{1}{\sec x}$.

$\frac{\cancel{\sin x}}{\cancel{\cos x}} \cdot \frac{1}{\cancel{\sin x}} \cdot \cos^{\cancel{2}} x =$

$\cos x =$

$\frac{1}{\sec x} = \frac{1}{\sec x}$ ✓

23. Simplify. $\cos\left(\frac{3\pi}{2} + \theta\right)$

$\cos\frac{3\pi}{2}\cos\theta - \sin\frac{3\pi}{2}\sin\theta$

$0 \cdot \cos\theta - (-1)\sin\theta = 0 + \sin\theta = \boxed{\sin\theta}$

24. Justify. $2\sin x(\sec x - \cos x) = 2\tan x - \sin 2x$

$2\sin x\left(\frac{1}{\cos x} - \cos x\right) =$

$2\sin x\cos x = \sin 2x$ (double angle identity)

$\frac{2\sin x}{\cos x} - 2\sin x\cos x = \rightarrow 2\tan x - \sin 2x =$ ✓

25. Solve $4\cos^2 x - 1 = 0$ on $0 \le x < 2\pi$.

* diff. of squares!

$(2\cos x + 1)(2\cos x - 1) = 0$

$2\cos x + 1 = 0$ $\quad$ $2\cos x - 1 = 0$

$\cos x = \frac{-1}{2}$ $\quad$ $\cos x = \frac{1}{2}$

$\boxed{x = \frac{2\pi}{3}, \frac{4\pi}{3} \quad x = \frac{\pi}{3}, \frac{5\pi}{3}}$

26. Solve $3\tan x - \sqrt{3} = 0$.

$\tan x = \frac{\sqrt{3}}{3}$

$x = \frac{\pi}{6} + \pi k$ $\quad$ ~~$x = \frac{7\pi}{6}$~~ already accounted for in 1st solution with $+\pi k$.

27. Solve $\cot^2\theta - 1 = 0$ on $0^\circ \le \theta < 180^\circ$.

$\cot^2\theta = 1$

$\cot\theta = \pm 1$

Where does $\frac{\cos\theta}{\sin\theta} = \pm 1$? 45° and 135°

28. Solve. $4\cos\theta\sin\theta + 2\cos\theta = 0$.

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ and $\frac{7\pi}{6}, \frac{11\pi}{6}$

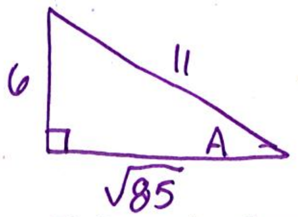
$2\cos\theta(2\sin\theta + 1) = 0$

$2\cos\theta = 0 \quad 2\sin\theta + 1 = 0$

$\cos\theta = 0 \quad \sin\theta = -\frac{1}{2}$

No restriction: $\frac{\pi}{2} + \pi k$, $\frac{7\pi}{6} + 2\pi k$, $\frac{11\pi}{6} + 2\pi k$

29. Determine $\sec 2A$ if $\sin A = \frac{6}{11}$ and angle A is in quadrant I.



$6^2 + x^2 = 11^2$

$x^2 = 85$

$x = \sqrt{85}$

$\cos A = \frac{\sqrt{85}}{11}$

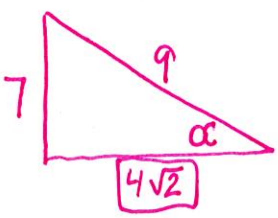
* run as cos 2A then flip.

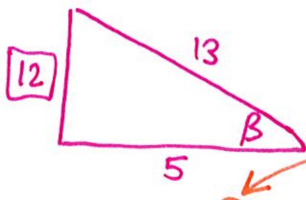
$\cos 2A = \cos^2 A - \sin^2 A$

$= \left(\frac{\sqrt{85}}{11}\right)^2 - \left(\frac{6}{11}\right)^2 = \frac{85}{121} - \frac{36}{121} = \boxed{\frac{49}{121}}$

$= \boxed{\frac{121}{49}}$

30. Determine the exact value of $\cos(\alpha - \beta)$ if $\sin\alpha = \frac{7}{9}$, $0 \le \alpha < \frac{\pi}{2}$, and $\cos\beta = \frac{5}{13}$, $\frac{3\pi}{2} < \beta < 2\pi$.





$49 + x^2 = 81$

$\cos\alpha = \frac{4\sqrt{2}}{9}$

$\sin\beta = \frac{-12}{13}$

$\cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\left(\frac{4\sqrt{2}}{9}\right)\left(\frac{5}{13}\right) + \left(\frac{7}{9}\right)\left(\frac{-12}{13}\right)$

$\frac{20\sqrt{2}}{117} - \frac{84}{117}$

$\boxed{\frac{20\sqrt{2} - 84}{117}}$