

Unit 5 - Matrices

What is the purpose of this unit?	EQ: How can we use matrices to solve real life problems?
What vocab do I need?	Vocabulary: matrix, determinant, elements, dimensions, scalar, inverse matrix, identity matrix
What is a determinant?	Determinant - a real number associated with a square matrix. The determinant of a matrix A is denoted by $\det A$ or $ A $.

A. Determinant of a 2 x 2 matrix

How do I find the determinant of a 2x2?	The determinant of a 2 x 2 matrix is the difference of the products of the elements on the diagonals. $\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb$
Example:	Ex. Evaluate the determinant of the matrix $\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$ $2(4) - (-1)(3)$ $8 + 3 = 11$

Ex #1 Evaluate each determinant

a) $\det \begin{bmatrix} 5 & 8 \\ 9 & 4 \end{bmatrix}$ b) $\det \begin{bmatrix} 3 & -4 \\ 7 & -2 \end{bmatrix}$

$5(4) - 9(8)$
 $20 - 72$
 -52

$3(-2) - (7)(-4)$
 $-6 + 28$
 22

B. Determinant of a 3 x 3 matrix

How do I find the determinant of a 3x3?	Rewrite the first two columns to the right of the determinant. Add the products of the leading diagonals and subtract from this the products of the opposite diagonals. $\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{vmatrix} a & b & c & a & b \\ d & e & f & d & e \\ g & h & i & g & h \end{vmatrix}$ $= (aei + bfg + cdh) - (gec + hfa + idb)$
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Example: Ex. Evaluate the determinant of the matrix

$$\begin{vmatrix} 1 & 2 & -1 & 1 & 2 \\ 0 & 3 & -2 & 0 & 3 \\ 4 & 1 & 5 & 4 & 1 \end{vmatrix}$$

$[15 - 16 + 0] - [-12 - 2 + 0]$
 $-1 - [-14]$
 13

Example: Ex. Evaluate:

$$\begin{vmatrix} -1 & 3 & 5 \\ 0 & x & 3 \\ -2 & 2 & x \end{vmatrix} = x - 4$$

$-1 \quad 3 \quad 5 \quad -1 \quad 3$
 $0 \quad x \quad 3 \quad 0 \quad x$
 $-2 \quad 2 \quad x \quad -2 \quad 2$

$[-x^2 - 18 + 0] - [-10x - 6 + 0] = x - 4$
 $-x^2 - 18 + 10x + 6 = x - 4$
 $0 = x^2 - 9x + 8$
 $0 = (x - 8)(x - 1)$
 $x = 1, 8$

C. Applications of Determinants

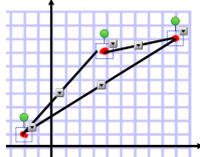
How can I use determinants? The determinant of a matrix can be used to find the area of a triangle. If (x_1, y_1) , (x_2, y_2) , and (x_3, y_3) are vertices of a triangle, the area of the triangle is:

$$\text{Area} = \pm \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

choose appropriate option

Example:

Ex. Find the area of the triangle given the points $(-2, 1)$, $(4, 7)$, $(9, 8)$.



$$\pm \frac{1}{2} \begin{vmatrix} -2 & 1 & 1 \\ 4 & 7 & 1 \\ 9 & 8 & 1 \end{vmatrix} = \pm \frac{1}{2} [14 + 9 + 32] - [63 - 16 + 4]$$

$$\pm \frac{1}{2} [27 - 51]$$

Area = $(-\frac{1}{2})(-24)$ need $(-\frac{1}{2})$ because of (-24)

$$= 12 \text{ u}^2$$

Vocabulary:

What is an identity matrix? **Identity matrix** - a square matrix with 1's along the leading diagonal and 0's elsewhere.

- An identity matrix is denoted using I .
- $AI = IA = A$.

What is an inverse? **Inverses** - the square matrices A and B are inverses of each other if their product (in both orders) is equal to an identity matrix. i.e. $AB=I$ and $BA=I$.

- Matrix A has an inverse iff $\det A \neq 0$
- The inverse of matrix A is denoted as A^{-1} .

A. Find the inverse of a 2 x 2 matrix

How do I find the inverse of a 2x2?

Given the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and assuming $ad - cb \neq 0$

$$A^{-1} = \frac{1}{|A|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad - cb} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Example: Find the inverse of each matrix and verify the ma

$$A = \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 6 & 2 \\ 9 & 3 \end{bmatrix}$$

B. Find the inverse of a 3 x 3 matrix

How do I find the inverse of a 3x3?

Use a calculator!

Ex. Find the inverse of $\begin{bmatrix} 4 & 0 & -1 \\ 6 & -2 & 0 \\ 3 & 1 & -4 \end{bmatrix}$

Verify they are inverses.