

<u>Unit 5 - Conic Sections</u>	
What is the purpose of this unit?	EQ: What is a conic section?
What vocab do I need?	Vocabulary: conic section, circle, tangent, ellipse, focus, foci, parabola, directrix, latus rectum, hyperbola, asymptote, 3D space, rectangular prism, sphere, plane.

<u>Circles</u>	
What is the purpose of this lesson?	EQ: How do I write the equation of a circle? How do I solve a system of equations containing a circle?

What is a conic section?	Vocabulary: Conic Sections - created by the intersection of a double cone and a plane. Types: circle, ellipse, parabola and hyperbola.
What is a circle?	Circle - the set of all points in a plane that are equidistant from a fixed point called the center. Standard equation of a circle with center at (h, k) and radius r: $(x - h)^2 + (y - k)^2 = r^2$ ↑ ↑ coeff must be 1

A. Write the equation of a circle

Ex: The point (6, 2) lies on a circle whose center is the origin. Write the standard form of the equation of the circle.

How do you write the equations of circles?

$r =$ ctr: (0,0)

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$r^2 = d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

$r^2 = (6-0)^2 + (2-0)^2 = 36 + 4 = 40$

$(x-h)^2 + (y-k)^2 = r^2$

$x^2 + y^2 = 40$

Ex: Write the standard equation for the circle. State the coordinates of the center and the radius and sketch a graph of the circle.

a. $2x^2 = 32 - 2y^2$

$2x^2 + 2y^2 = 32$

$x^2 + y^2 = 16$

ctr: (0,0)

$r = 4$

$r^2 = 16$

$r = 4$

b. $x^2 + y^2 - 8x + 7 = 0$

$x^2 - 8x + 16 + y^2 = -7 + 16$

$(x-4)^2 + y^2 = 9$

ctr: (4,0)

$r = 3$

$ax^2 + bx + c = y$

$C_{new} = \left(\frac{b}{2a}\right)^2$

$C_{new} = \left(\frac{-8}{2}\right)^2 = (-4)^2 = 16$

c. $x^2 + y^2 + 22x - 2y = -120$

$$x^2 + 22x + \underline{121} + y^2 - 2y + \underline{1} = -120$$

$C_1 = \left(\frac{-22}{2}\right) = (-11) \Rightarrow 121$ $C_2 = \left(\frac{-2}{2}\right) = (-1) \Rightarrow 1$

$$(x+11)^2 + (y-1)^2 = 2$$

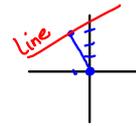
ctr: $(-11, 1)$
 $r = \sqrt{2}$

B. Write the equation of a line tangent to a circle:

How do you find the equation of a line tangent to the circle?

Ex: Find an equation of the line tangent to the circle $x^2 + y^2 = 10$ at $(-1, 3)$.

ctr: $(0, 0)$
 $r \perp$ tangent



$$m_r = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 0}{-1 - 0} = -3$$

$y - y_1 = m(x - x_1)$ $m_t = +1/3$ ← opp/opp

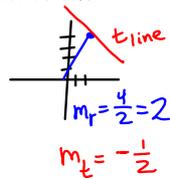
$y = m(x - x_1) + y_1$

$y = \frac{1}{3}(x - (-1)) + 3$

$y = \frac{1}{3}x + \frac{1}{3} + 3$

$y = \frac{1}{3}x + 3\frac{1}{3}$

Ex: Find an equation of the line tangent to the circle $x^2 + y^2 = 20$ at $(2, 4)$.



$y = m(x - x_1) + y_1$

$y = -\frac{1}{2}(x - 2) + 4$

$y = -\frac{1}{2}x + 1 + 4$

$y = -\frac{1}{2}x + 5$

C-1) $r = 8$ ctr: $(3, -2)$

$(x - 3)^2 + (y + 2)^2 = 64$

C-2) $r = 9/25$ ctr: $(0, -1)$

$x^2 + (y + 1)^2 = \frac{81}{625}$

C-3) $d = 3/4$ ctr: $(0, 0)$

$r = \frac{3}{8}$

$x^2 + y^2 = \frac{9}{64}$

D-1) $6x^2 + 6y^2 = 48$

$x^2 + y^2 = 8$

ctr: $(0, 0)$

$r = 2\sqrt{2}$
 ≈ 2.8

