PreCalculus

Angle Formulas and Solving Trig Equations

Name Answer key

Practice 1

Part I: Evaluate the following trig ratios using the appropriate sum or difference formula.

1. $\sin(105°)$
2. $\cos(285°)$
3. $\tan(-75°)$
4. $\sin(-\frac{\pi}{12})$
5. $\cos(\frac{11\pi}{12})$
6. $\tan(\frac{5\pi}{12})$

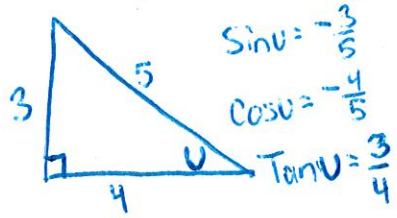
$\frac{\pi}{4} - \frac{\pi}{3}$

Part II: Evaluate the following trig ratios using the double or half angle formula.

1. $\sin(165°)$
2. $\cos(120°)$
3. $\tan(\frac{5\pi}{8})$

Part III. Evaluate the following trig ratios if $\sin u = -\frac{3}{5}$ where $\pi < u < \frac{3\pi}{2}$ and $\tan v = \frac{-24}{7}$ where $\frac{\pi}{2} < v < \pi$.

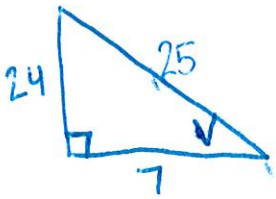
Draw 2 triangles and label sine, cosine and tangent for each.



$\sin u = -\frac{3}{5}$

$\cos u = -\frac{4}{5}$

$\tan u = \frac{3}{4}$



$\sin v = \frac{24}{25}$

$\cos v = \frac{-7}{25}$

$\tan v = \frac{-24}{7}$

1. $\cos u$
2. $\sin(2u)$
3. $\tan(\frac{u}{2})$
4. $\sin v$
5. $\cos(2v)$
6. $\tan(\frac{v}{2})$
7. $\cos(u+v)$
8. $\sin(u+v)$
9. $\tan(u+v)$
10. $\cos(v-u)$
11. $\sin(u-v)$
12. $\tan(v-u)$
13. $\sec(2u)$
14. $\csc(v-u)$
15. $\cot(\frac{v}{2})$

PreCalc Angle Formulas and Solving Trig Equations.

Part I

1. $\sin(105^\circ) = \sin(60^\circ + 45^\circ) = \sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ$
$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$

2. $\cos(285^\circ) = \cos(150^\circ + 135^\circ) = \cos 150^\circ \cos 135^\circ - \sin 150^\circ \sin 135^\circ$
$= \left(-\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{6} - \sqrt{2}}{4}$

3. ~~$\tan(45^\circ - 120^\circ) = \tan 45^\circ - \tan 120^\circ$~~
~~$1 + \tan 45^\circ \tan 120^\circ$~~

$$\tan(45^\circ - 120^\circ) = \frac{\tan 45^\circ - \tan 120^\circ}{1 + \tan 45^\circ \tan 120^\circ} = \frac{1 - (-\sqrt{3})}{1 + (-\sqrt{3})} = \frac{1 + \sqrt{3}}{1 - \sqrt{3}}$$

4. $\sin\left(\frac{\pi}{4} - \frac{\pi}{3}\right) = \sin\frac{\pi}{4}\cos\frac{\pi}{3} - \cos\frac{\pi}{4}\sin\frac{\pi}{3}$
$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{2} - \sqrt{6}}{4}$

5. $\cos\left(\frac{3\pi}{4} + \frac{\pi}{6}\right) = \cos\frac{3\pi}{4}\cos\frac{\pi}{6} - \sin\frac{3\pi}{4}\sin\frac{\pi}{6}$
$= \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) = \frac{-\sqrt{6} - \sqrt{2}}{4}$

6. $\tan\left(\frac{\pi}{4} + \frac{\pi}{6}\right) = \frac{\tan\frac{\pi}{4} + \tan\frac{\pi}{6}}{1 - \tan\frac{\pi}{4}\tan\frac{\pi}{6}} = \frac{1 + \frac{\sqrt{3}}{3}}{1 - \frac{\sqrt{3}}{3}} = \frac{\frac{3+\sqrt{3}}{3}}{\frac{3-\sqrt{3}}{3}} = \frac{3 + \sqrt{3}}{3 - \sqrt{3}}$

Part II

1. $\sin\left(\frac{1}{2}(330^\circ)\right) = \sqrt{\frac{1-\cos 330^\circ}{2}} = \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2-\sqrt{3}}{2}} = \sqrt{\frac{2-\sqrt{3}}{4}}$

$= \boxed{\frac{\sqrt{2-\sqrt{3}}}{2}}$

2. $\cos(2(60^\circ)) = \cos^2 60^\circ - \sin^2 60^\circ$

$= \frac{1}{4} - \frac{3}{4} = \boxed{-\frac{1}{2}}$

3. $\tan\left(\frac{1}{2}\left(\frac{5\pi}{6}\right)\right) = \frac{\sin\frac{5\pi}{6}}{1-\cos\frac{5\pi}{6}} = \frac{\frac{1}{2}}{1-\left(-\frac{\sqrt{3}}{2}\right)} = \frac{1}{2}\cdot\frac{2}{2+\sqrt{3}} = \boxed{\frac{1}{2+\sqrt{3}}}$

Part III

1. $\boxed{-\frac{4}{5}}$

2. $2\sin u\cos u = 2\left(-\frac{3}{5}\right)\left(-\frac{4}{5}\right) = \boxed{\frac{24}{25}}$

3. $\frac{\sin u}{1+\cos u} = \frac{-\frac{3}{5}}{1+\left(-\frac{4}{5}\right)} = -\frac{3}{5}\cdot\frac{5}{1} = \boxed{-3}$

4. $\boxed{\frac{24}{25}}$

5. $\cos^2 v - \sin^2 v = \frac{49}{625} - \frac{576}{625} = \boxed{\frac{-527}{625}}$

6. $\frac{\sin v}{1-\cos v} = \frac{24}{25}\cdot\frac{25}{32} = \boxed{\frac{3}{4}}$

7. $\cos u\cos v - \sin u\sin v = \left(-\frac{4}{5}\right)\left(-\frac{7}{25}\right) - \left(-\frac{3}{5}\right)\left(\frac{24}{25}\right)$

$= \frac{28}{125} + \frac{72}{125} = \frac{100}{125} = \boxed{\frac{4}{5}}$

8. $\sin u\cos v + \cos u\sin v = \left(-\frac{3}{5}\right)\left(-\frac{7}{25}\right) + \left(-\frac{4}{5}\right)\left(\frac{24}{25}\right)$

$= \frac{21}{125} - \frac{96}{125} = \frac{-75}{125} = \boxed{-\frac{3}{5}}$

9. $\dfrac{\tan u + \tan v}{1 - \tan u \tan v} = \dfrac{\frac{3}{4} + \frac{-24}{7}}{1 + \frac{72}{28}} = \dfrac{\frac{-75}{28}}{\frac{100}{28}} = \dfrac{-75}{28} \cdot \dfrac{28}{100} = \boxed{\dfrac{-3}{4}}$

10. $\cos v \cos u + \sin v \sin u = \left(\frac{-7}{25}\right)\left(\frac{-4}{5}\right) + \left(\frac{24}{25}\right)\left(\frac{-3}{5}\right) = \frac{28}{125} + \frac{-72}{125} = \boxed{\frac{-44}{125}}$

11. $\sin u \cos v - \cos u \sin v = \left(\frac{-3}{5}\right)\left(\frac{-7}{25}\right) - \left(\frac{-4}{5}\right)\left(\frac{24}{25}\right) = \frac{21}{125} + \frac{96}{125} = \boxed{\frac{117}{125}}$

12. $\dfrac{\tan v - \tan u}{1 + \tan v \tan u} = \dfrac{\left(\frac{-24}{7}\right) - \left(\frac{3}{4}\right)}{1 + \left(\frac{-24}{7}\right)\left(\frac{3}{4}\right)} = \dfrac{\frac{-117}{28}}{\frac{-44}{28}} = \dfrac{-117}{28} \cdot \dfrac{28}{-44} = \boxed{\dfrac{117}{44}}$

13. $* \cos(2u) = \cos^2 u - \sin^2 u = \left(\frac{16}{25}\right) - \left(\frac{9}{25}\right) = \frac{7}{25}$

$* \sec(2u) = \boxed{\frac{25}{7}}$

14. $* \sin(v-u) = \sin v \cos u - \cos v \sin u = \left(\frac{24}{25}\right)\left(\frac{-4}{5}\right) - \left(\frac{-7}{25}\right)\left(\frac{-3}{5}\right)$

$= \frac{-96}{125} - \frac{21}{125} = \frac{-117}{125}$

$* \csc(v-u) = \boxed{\frac{-125}{117}}$

15. * See #6 and reciprocal! $\boxed{\frac{4}{3}}$

Part IV

1. $\cos x = \frac{-1}{2}$

a) $x = \frac{2\pi}{3}, \frac{4\pi}{3}$

b) $x = \frac{2\pi}{3} + 2\pi k$

$x = \frac{4\pi}{3} + 2\pi k$

2. $\csc^2 x = \frac{4}{3}$

$\csc x = \pm \frac{2}{\sqrt{3}}$

$\sin x = \pm \frac{\sqrt{3}}{2}$

a) $x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

b) $x = \frac{\pi}{3} + \pi k$

$x = \frac{2\pi}{3} + \pi k$

3. $\cos^2 x = \frac{3}{4}$

$\cos x = \pm \frac{\sqrt{3}}{2}$

a) $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

b) $x = \frac{\pi}{6} + \pi k$

$x = \frac{5\pi}{6} + \pi k$

4. $3\tan^2 x - 1 = 0$

$\tan^2 x = \frac{1}{3}$

$\tan x = \pm \frac{1}{\sqrt{3}}$

a) $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

b) $x = \frac{\pi}{6} + \pi k$

$x = \frac{5\pi}{6} + \pi k$

$\tan^2 x = 3$

$\tan x = \pm\sqrt{3}$

$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

$x = \frac{\pi}{3} + \pi k$

$x = \frac{2\pi}{3} + \pi k$

Part V

5. $3\tan^3 x - \tan x = 0$

$\tan x(3\tan^2 x - 1) = 0$

$\tan x = 0$ $\quad \tan^2 x = \frac{1}{3}$

$\tan x = \pm \frac{\sqrt{3}}{3}$

a) $x = \pi, 0$ $\quad x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

b) $x = \pi + \pi k$ (or πk)

$x = \frac{\pi}{6} + \pi k$

$x = \frac{5\pi}{6} + \pi k$

6. $2\sin x + \frac{1}{\sin x} = 0$

$\frac{2\sin^2 x + 1}{\sin x} = 0$

a) $2\sin^2 x + 1 = 0$

$\sin^2 x = \frac{-1}{2}$

no solution!

7. $\frac{x}{2} = \frac{\pi}{4}$ $\quad \frac{x}{2} = \frac{7\pi}{4}$

a) $\boxed{x = \frac{\pi}{2}}$ $\quad \cancel{x = \frac{7\pi}{2}}$ more than 2π

b) $x = \frac{\pi}{2} + 4\pi k$

8. $\frac{2}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} - 3 = 0$

$\frac{2 + \sin^2 x - 3\cos^2 x}{\cos^2 x} = 0$

$\frac{2 + 1 - \cos^2 x - 3\cos^2 x}{\cos^2 x} = 0$

$\frac{3 - 4\cos^2 x}{\cos^2 x} = 0$

$\cos^2 x = \frac{3}{4}$

$\cos x = \pm\frac{\sqrt{3}}{2}$

a) $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

b) $x = \frac{\pi}{6} + \pi k$

$x = \frac{5\pi}{6} + \pi k$

9. $2x = \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{10\pi}{3}, \frac{11\pi}{3}$

a) $x = \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{5\pi}{3}, \frac{11\pi}{6}$

b) $x = \frac{2\pi}{3} + \pi k$

$x = \frac{5\pi}{6} + \pi k$

10. $3x = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}, \frac{17\pi}{4}, \frac{21\pi}{4}$

a) $x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{9\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{21\pi}{12}$

b) $x = \frac{\pi}{12} + \pi k$

$x = \frac{5\pi}{12} + \pi k$

$x = \frac{9\pi}{12} + \pi k$

Part VII

11. $x = 1.46$

$x = 4.82$

12. $x = .79$ $x = 5.76$

$x = 2.36$

$x = 3.67$

$x = 3.93$

$x = 5.50$