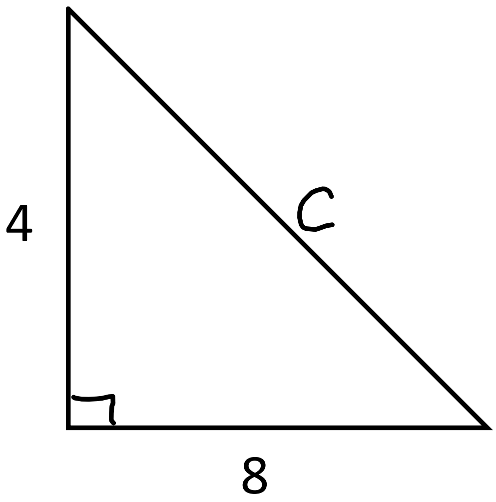
Find the length of the missing side of the triangle:



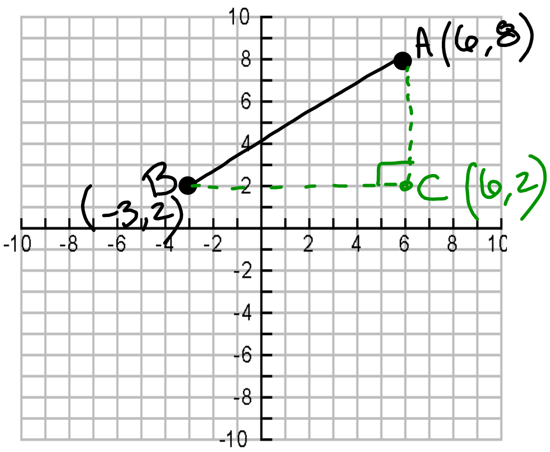
$c^2 = 4^2 + 8^2$

$c = \sqrt{4^2 + 8^2}$

$c = \sqrt{16+64}$

$c = \sqrt{80}$

$\boxed{c = 4\sqrt{5}}$



$AB = ?$

$AC = (8-2) = 6$

$BC = (6-(-3)) = 9$

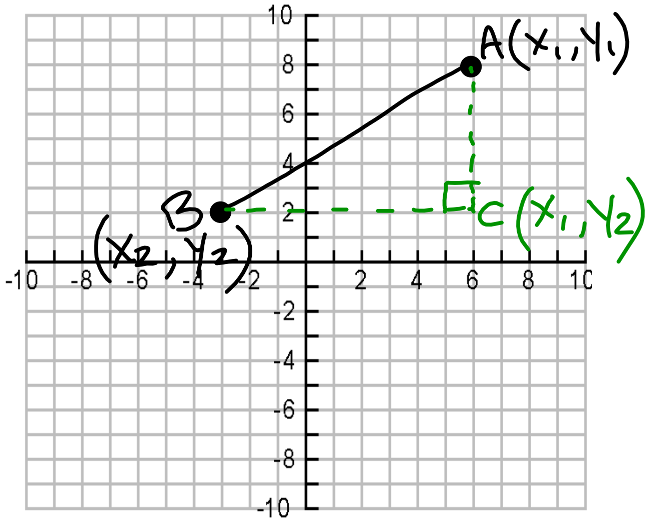
$AB^2 = AC^2 + BC^2$

$AB^2 = 6^2 + 9^2$

$AB = \sqrt{6^2 + 9^2}$

$= \sqrt{36+81}$

$= \sqrt{117} = \sqrt{9 \cdot 13} = \boxed{3\sqrt{13}}$



$AB = ?$

$AC = (y_1 - y_2)$

$BC = (x_1 - x_2)$

$$AB^2 = AC^2 + BC^2$$

$$AB^2 = (y_1 - y_2)^2 + (x_1 - x_2)^2$$

$$AB = \sqrt{(y_1 - y_2)^2 + (x_1 - x_2)^2}$$

<u>THE DISTANCE FORMULA</u>

Given two points, $A(x_1, y_1)$ and $B(x_2, y_2)$, the distance between them can be found using the formula:

$$AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

If $A(2, 3)$ and $B(7, 15)$, find AB

$x_1 \; y_1 \qquad x_2 \; y_2$

$$AB = \sqrt{(2-7)^2 + (3-15)^2}$$

$$= \sqrt{(-5)^2 + (-12)^2}$$

$$= \sqrt{25 + 144}$$

$$= \sqrt{169} = \boxed{13}$$

If $A(-9, -2)$ and $B(x, 5)$, find x if $AB = 7$

$$7 = \sqrt{(-9-x)^2 + (-2-5)^2}$$

$$7^2 = \sqrt{(-9-x)^2 + 49}^{\,2}$$

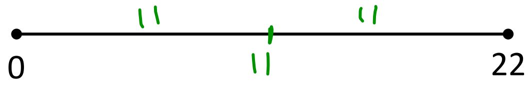
$$49 = (-9-x)^2 + 49$$

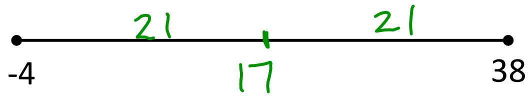
$$\sqrt{0} = \sqrt{(-9-x)^2}$$

$$0 = -9 - x$$

$$9 = -x \qquad \boxed{x = -9}$$

How can we find the point that divides a segment in half?





Formula: $x_1 + \frac{1}{2}(x_2 - x_1)$

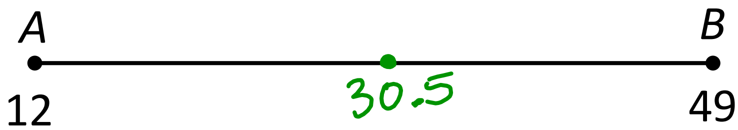
$$x_1 + \frac{1}{2}x_2 - \frac{1}{2}x_1$$

$$\frac{1}{2}x_1 + \frac{1}{2}x_2$$

$$\frac{1}{2}(x_1 + x_2)$$

$$\frac{x_1 + x_2}{2}$$

Find the coordinate of the point that is ½ of the way from *A* to *B*



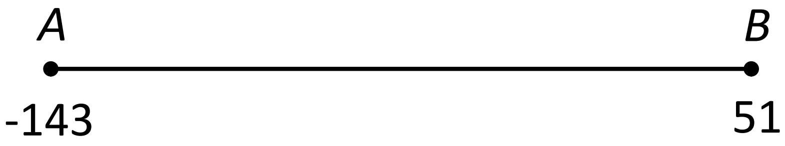
$$x_1 + \frac{1}{2}(x_2 - x_1)$$

$$12 + \frac{1}{2}(49 - 12)$$

$$12 + \frac{37}{2}$$

$$\frac{24}{2} + \frac{37}{2} = \boxed{\frac{61}{2}}$$

Find the coordinate of the point that is ½ of the way from *A* to *B*



$$x_1 + \frac{1}{2}(x_2 - x_1)$$

$$-143 + \frac{1}{2}(51 - (-143))$$

$$-143 + \frac{1}{2}(194)$$

$$-143 + 97$$

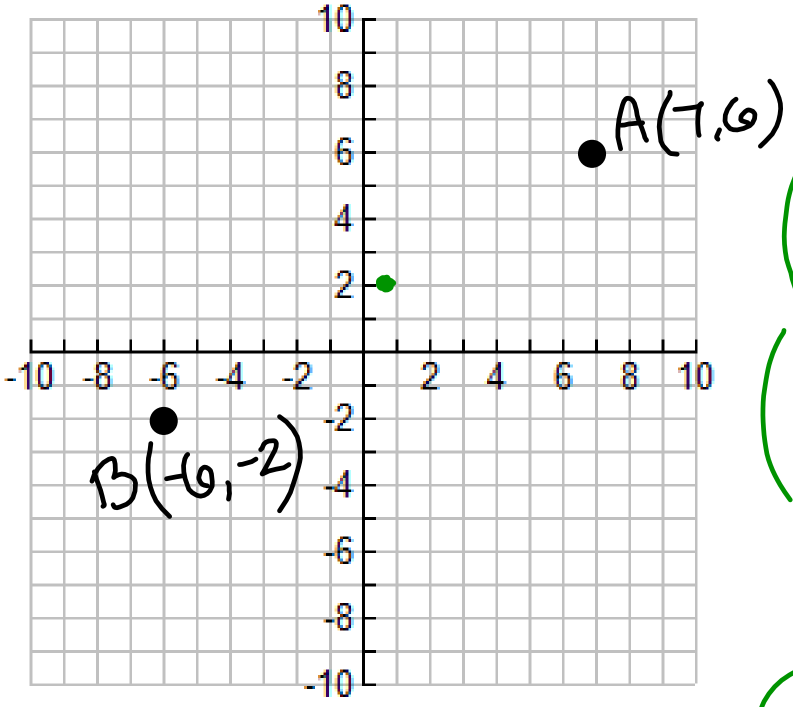
$$\boxed{-46}$$

THE MIDPOINT FORMULA

Given two points, $A(x_1, y_1)$ and $B(x_2, y_2)$, the MIDPOINT of $\overline{AB}$ can be found using the formula:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Find the coordinates of the midpoint of $\overline{AB}$



$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$\left(\frac{7+(-6)}{2}, \frac{6+(-2)}{2}\right)$$

$$\left(\frac{1}{2}, \frac{4}{2}\right)$$

$$\left(\frac{1}{2}, 2\right)$$

If the midpoint of $\overline{AB}$ falls at (7, 9), and point A is located at (-2, 4), then find the coordinates of B

$$\text{midpt} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$(7, 9) = \left(\frac{-2+x}{2}, \frac{4+y}{2}\right)$$

$$7 = \frac{-2+x}{2}$$

$$14 = -2 + x$$

$$\boxed{16 = x}$$

$$9 = \frac{4+y}{2}$$

$$18 = 4 + y$$

$$\boxed{14 = y}$$

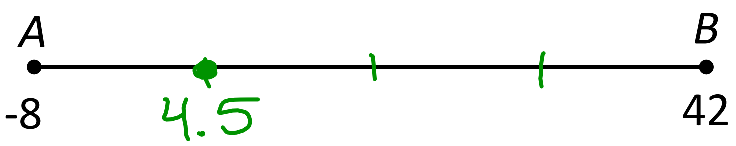
$$\therefore B(16, 14)$$

What if we want to partition the segment into thirds, or fourths, etc?
(ie, some ratio other than one-half)

$$x_1 + r(x_2 - x_1)$$

where *r* represents the desired ratio

Find the coordinate of the point that is ¼ of the way from *A* to *B*



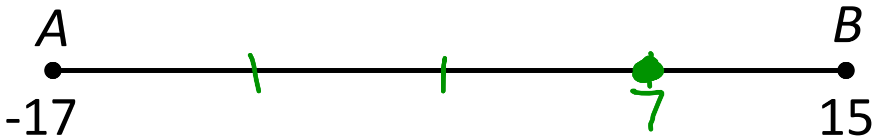
$$x_1 + \frac{1}{4}(x_2 - x_1)$$

$$-8 + \frac{1}{4}(42 - (-8))$$

$$-8 + \frac{1}{4}(50)$$

$$-8 + \frac{25}{2} = \frac{-16}{2} + \frac{25}{2} = \boxed{\frac{9}{2}}$$

Find the coordinate of the point that is ¾ of the way from *A* to *B*

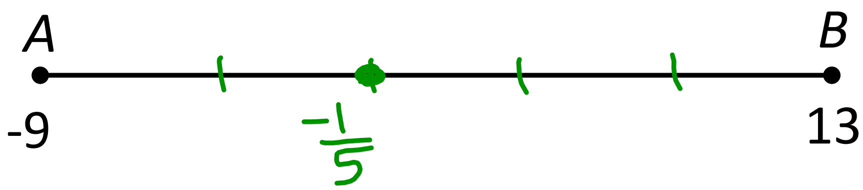


$$-17 + \frac{3}{4}\left(15-(-17)\right)$$

$$-17 + \frac{3}{4}(32)$$

$$-17 + 24 = \boxed{7}$$

Find the coordinate of the point that is ⅖ of the way from *A* to *B*

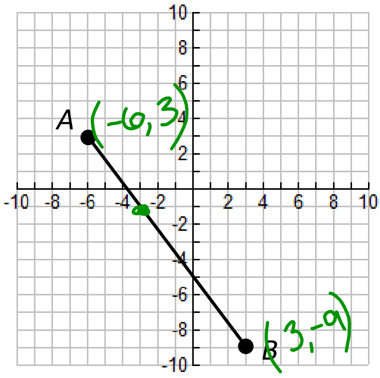


$$\boxed{-\frac{1}{5}}$$

What if the segment is in the coordinate plane?

$$\left(x_1 + r\left(x_2 - x_1\right),\ y_1 + r\left(y_2 - y_1\right)\right)$$

Find the coordinates of the point that is $2/3$ of the way from $\boxed{B \text{ to } A}$



x's : $3 + \frac{2}{3}(-6-3)$

$3 + \frac{2}{3}(-9)$

$3 - 6 = \boxed{-3}$

y's : $-9 + \frac{2}{3}(3-(-9))$

$-9 + \frac{2}{3}(12)$

$-9 + 8 = \boxed{-1}$

$(-3, -1)$

Point P lies on $\overline{AB}$, and the ratio of AP to PB is 3 to 2. If $A(3, 4)$ and $B(6, 10)$, then find the coordinates of P.

$A(3,4)$ —3— P —2— $B(6,10)$

$r = \frac{3}{5}$

$x_1 + \frac{3}{5}(x_2 - x_1)$

$3 + \frac{3}{5}(6-3)$

$3 + \frac{9}{5} = \frac{15}{5} + \frac{9}{5} = \boxed{\frac{24}{5}}$

$y_1 + \frac{3}{5}(10-4)$

$4 + \frac{3}{5}(6)$

$4 + \frac{18}{5} = \frac{20}{5} + \frac{18}{5} = \boxed{\frac{38}{5}}$

$\therefore P\left(\frac{24}{5}, \frac{38}{5}\right)$

Point *P* lies on $\overline{AB}$, and the ratio of *AP* to *PB* is 4 to 1. If *A*(1, 3) and *B*(8, 4), then find the coordinates of *P*.

A ——4——•—1— B

$r = \frac{4}{5}$

$1 + \frac{4}{5}(8-1)$

$1 + \frac{4}{5}(7)$

$1 + \frac{28}{5} = \boxed{\frac{33}{5}} = x$

$3 + \frac{4}{5}(4-3)$

$3 + \frac{4}{5} = \boxed{\frac{19}{5}} = y$

$\therefore P\left(\frac{33}{5}, \frac{19}{5}\right)$

Point *P* lies on $\overline{AB}$, and the ratio of *AP* to *PB* is 3 to 7. If *A*(-2, 1) and *B*(4, 9), then find the coordinates of *P*.

$r = \frac{3}{10}$

$\therefore P\left(-\frac{1}{5}, \frac{17}{5}\right)$

Point $P(3, -12)$ lies on $\overline{AB}$, and the ratio of AP to PB is 1 to 4. If A is at $(8, 16)$, then find the coordinates of B.

A $(8, 16)$ —1— $P(3, -12)$ —4— B (x, y)

$r = \frac{1}{5}$

$3 = 8 + \frac{1}{5}(x-8)$

$-5 = \frac{1}{5}(x-8)$

$-25 = x-8$

$\boxed{-17 = x}$

$-12 = 16 + \frac{1}{5}(y-16)$

$-28 = \frac{1}{5}(y-16)$

$-140 = y-16$

$\boxed{-124 = y}$

$\therefore B(-17, -124)$

Point $P(-8, 13)$ lies on $\overline{AB}$, and the ratio of AP to PB is 2 to 7. If B is at $(11, -24)$, then find the coordinates of A.

A (x, y) —2— $P(-8, 13)$ —7— B $(11, -24)$

$r = \frac{2}{9}$

$-8 = x + \frac{2}{9}(11-x)$

$-8 = x + \frac{22}{9} - \frac{2x}{9}$

$-72 = 9x + 22 - 2x$

$-72 = 7x + 22$

$-94 = 7x$

$\boxed{x = \frac{-94}{7}}$

$13 = y + \frac{2}{9}(-24-y)$

$13 = y + \left(\frac{-48}{9}\right) - \frac{2y}{9}$

$117 = 9y - 48 - 2y$

$165 = 7y$

$\boxed{y = \frac{165}{7}}$

$\therefore A\left(\frac{-94}{7}, \frac{165}{7}\right)$